JOURNAL OF REPRODUCIBLE STATISTICS
FAIR Press Journals · jrepstat.fairpressjournals.com · ISSN 3043-2790
Vol. 1 (2026), Art. 14
10.68365/jrepstat.2026.14

RESEARCH ARTICLE · SIMULATION STUDY · FOUNDING ARTICLE

James–Stein Estimation of Structural Equation Models With Latent Variables: A Case Study With lavaan

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ARTICLE INFO

Keywords
structural equation modeling
James–Stein estimation
model-implied instrumental variables
delta method
simulation

History
Received 28 Oct 2026
Final version 6 Nov 2026
Accepted 14 Nov 2026
Version 1

Handling editor
Matthias Templ

ABSTRACT

This article presents scaling (JS) and aggregated (JSA) James–Stein estimation of structural equation models with latent variables, based on their implementation in the R package lavaan. Starting from a worked example, we describe the estimation of factor loadings and latent regression slopes equation by equation, followed by the estimation of (residual) variance, covariance, and intercept parameters. We also provide analytical standard errors for JS and JSA using the delta method, accounting for uncertainty across all estimation stages, which were not available before. Throughout, model-implied instrumental-variable (MIIV) estimation serves as a point of comparison using a common notation. A simulation study compares the standard error accuracy of JS and JSA with that of MIIV and maximum likelihood (ML) for a correctly specified model with normally distributed data. Under these conditions, the standard errors of JS and JSA perform comparably to those of MIIV and ML.

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Result	Reported results reproduced. Simulation rerun from the seed on two Linux images: all 5,805 table and 49 text values match, replications agree with the archive to 3e-6.	✓ *measured*
Entry point	`cd code/felipelfv-simulation_js-62b9f09 && Rscript run_all.R / quarto render manuscript.qmd`	✓ *executed*
Environment	R 4.6.0, lavaan 0.7.3.3364, simhelpers 0.3.1, ggplot2 4.0.3, knitr 1.51; the authors' Nix image (rstats-on-nix 2026-05-11) and rocker/r-ver:4.6.0 on colima, linux/aarch64	✓ *measured*
Determinism	Two runs byte-identical on Linux aarch64. Full simulation run twice in a container with one CPU.	✓ *measured*
Archive	`james-stein-estimation-of-structural-equation-models-with-latent-variables-a-case-study-with-lavaan-v1-repro.zip` · reviewed bundle: the author's archived release `https://doi.org/10.5281/zenodo.23946236`, SHA-256 `2b8b4cbba4525e6c51ed59767a3e845cc9eeec4889ed9f3fa60ac472fa694928`	✓ *computed*
Code	`10.5281/zenodo.23730879`	*declared*
Data	Open.	*declared, route checked*
AI use	Language editing and code assistance with Claude Opus 5.5 (Anthropic).	*declared*

Structural equation models (SEMs) with latent variables are usually estimated by maximum likelihood (ML), which estimates all parameters jointly. Although ML has desirable asymptotic properties when its

CITE AS Fontana Vieira, F., & Rosseel, Y. (2026). James–Stein Estimation of Structural Equation Models With Latent Variables: A Case Study With lavaan. Journal of Reproducible Statistics, 1, 14. https://doi.org/10.68365/jrepstat.2026.14

assumptions hold (e.g, Browne, 1984), difficulties may arise when it is applied in practice. For instance, a misspecification in one equation can bias estimates in other equations (Bollen et al., 2007; Bollen, 2019; Dhaene & Rosseel, 2023; Kaplan, 1988; see also Jin et al., 2021, for diagonally weighted least squares), and the iterative optimization can fail to converge or end in inadmissible solutions, especially in small samples (Bollen, 2019; Dhaene & Rosseel, 2023). Equation-by-equation estimators offer an alternative (e.g., Bollen, 1996; Burghgraeve et al., 2021). With these approaches, factor loadings and latent regression slopes are estimated one equation at a time and in closed form, which may limit the propagation of misspecification across equations (Bollen et al., 2018; Burghgraeve et al., 2021) and avoids iterative optimization for this part of the estimation.

A well-known equation-by-equation estimator in the SEM literature is model-implied instrumental-variable (MIIV) estimation[1] (Bollen, 1996, 2019; Bollen & Bauer, 2004). MIIV replaces each latent variable by its scaling indicator, which turns every equation into one between observed variables. In this case, the measurement error of the scaling indicator becomes part of the error term of the equation and is correlated with the predictors. Ordinary least squares is therefore inconsistent, and MIIV corrects for the resulting measurement error with instruments implied by the model, estimating each equation by two-stage least squares.

This article focuses on an alternative equation-by-equation estimator, James–Stein estimation, proposed for SEMs with latent variables in Burghgraeve et al. (2021), which replaces each latent predictor not by its scaling indicator but by an estimate of its conditional expectation given observed proxies and then applies least squares. The conditional expectation is uncorrelated with the error term of the equation by design, which removes the need for instruments. This conditional expectation is estimated using James–Stein type shrinkage (Efron & Morris, 1973, 1975; James & Stein, 1961), and the approach follows the principle of regression calibration in errors-in-variables regression (Carroll et al., 2006; Fuller, 1987; Whittemore, 1989). James–Stein estimation comes in a scaling version (JS), which uses the scaling indicators as proxies, and an aggregated version (JSA), which uses reliability weighted sums of indicators. JSA was found to outperform ML and MIIV in terms of mean squared error under the small sample conditions examined (Burghgraeve et al., 2021), and both versions were further evaluated for confirmatory factor models by Dhaene & Rosseel (2024). Throughout this article, MIIV serves as a point of comparison. Placing the two side by side makes clear what JS and JSA share with MIIV and where they depart from it.

Beyond point estimation, the usefulness of JS and JSA for statistical inference depends on the accuracy of their standard errors. Burghgraeve et al. (2021) derived a sampling variance expression for latent regression estimates that accounts for uncertainty in the estimated reliability adjustment, providing a basis for analytical standard errors for those estimates. However, they did not develop a complete analytical standard error calculation for all model parameters and used bootstrap standard errors for both JS and JSA in their empirical evaluations. For MIIV, by contrast, standard errors are well established. Conventional standard errors for factor loadings and latent regression slopes follow from two-stage least squares (Bollen, 1996). Standard errors for these same estimates can also be obtained by the delta method, using derivatives with respect to the sample moments, for which Fisher & Bollen (2020) provided explicit analytical expressions. These derivatives also propagate uncertainty in the estimated loadings and slopes into the standard errors of subsequently estimated variance and covariance parameters. Standard errors for these latter parameters were derived under unweighted least squares by Bollen & Maydeu-Olivares (2007) and extended to general weight matrices by Jin et al. (2021). More recently, Maydeu-Olivares et al. (2026) compared subsequent variance and covariance estimation procedures and the accuracy of their standard errors.

For JS and JSA, this gap has recently been closed in software. The R package lavaan (Rosseel, 2012) now provides analytical standard errors for both by applying the delta method to the complete estimation procedure. In addition, its JS and JSA implementations, initially restricted to confirmatory factor models, have been extended to more general models, and the package also includes MIIV estimation. Yet neither

[1]Throughout this article, MIIV refers to model-implied instrumental-variable two-stage least squares (MIIV-2SLS) estimation (Bollen, 1996, 2019). Other estimators use the same model-implied instruments, such as MIIV-GMM (Bollen et al., 2014), which can estimate several equations jointly, but these are not considered here.

these analytical standard errors nor the extended implementations have been presented or systematically evaluated.

0.1 *Present Study and Contributions*

This article makes three contributions. First, it describes JS and JSA, as implemented in lavaan, and conceptually compares them with MIIV estimation, starting from a worked example. Second, it develops and illustrates a complete delta method calculation of standard errors for JS and JSA, accounting for uncertainty across all estimation stages. Third, it evaluates their standard error accuracy in a simulation study comparing JS and JSA with MIIV and ML.

The remainder of the article introduces the model and notation, presents the estimators for all parameters together with their analytical standard errors, and reports the simulation study and its results, followed by a discussion.

1 Model, Notation, and Parameterization

Following the all-y LISREL notation (Jöreskog & Sörbom, 1996, submodel 3B, Chapter 6; Rosseel & Vidal, 2025), let $\mathbf{y} \in \mathbb{R}^p$ contain the continuous observed variables and $\boldsymbol{\eta} \in \mathbb{R}^q$ the latent variables. The measurement and latent variable (or structural) models are respectively

$$\mathbf{y} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta} + \boldsymbol{\epsilon}, \qquad \boldsymbol{\eta} = \boldsymbol{\alpha} + \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\zeta},$$

where $\boldsymbol{\nu} \in \mathbb{R}^p$ contains the measurement intercepts, $\boldsymbol{\Lambda} \in \mathbb{R}^{p\times q}$ contains the factor loadings, $\boldsymbol{\alpha} \in \mathbb{R}^q$ contains the structural intercepts, and $\mathbf{B} \in \mathbb{R}^{q\times q}$ contains the latent regression slopes, that is, the regression slopes among the latent variables. The measurement errors $\boldsymbol{\epsilon} \in \mathbb{R}^p$ and structural disturbances $\boldsymbol{\zeta} \in \mathbb{R}^q$ satisfy $\mathbb{E}(\boldsymbol{\epsilon}) = \mathbf{0}$ and $\mathbb{E}(\boldsymbol{\zeta}) = \mathbf{0}$. Let $\mathbb{S}_+^m$ and $\mathbb{S}_{++}^m$ denote the sets of symmetric positive semidefinite and positive definite $m \times m$ matrices, respectively. Assume

$$\mathrm{Var}(\boldsymbol{\epsilon}) = \boldsymbol{\Theta} \in \mathbb{S}_+^p, \qquad \mathrm{Var}(\boldsymbol{\zeta}) = \boldsymbol{\Psi} \in \mathbb{S}_+^q, \qquad \mathrm{Cov}(\boldsymbol{\epsilon}, \boldsymbol{\zeta}) = \mathbf{0}_{p\times q}.$$

Provided that $\mathbf{I}_q - \mathbf{B}$ is nonsingular, the latent and observed mean vectors are

$$\boldsymbol{\mu}_\eta = \left(\mathbf{I}_q - \mathbf{B}\right)^{-1}\boldsymbol{\alpha} \in \mathbb{R}^q, \qquad \boldsymbol{\mu}_y = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta \in \mathbb{R}^p,$$

where $\mathbf{I}_q$ is the $q \times q$ identity matrix. The implied observed covariance matrix is

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\left(\mathbf{I}_q - \mathbf{B}\right)^{-1}\boldsymbol{\Psi}\left(\mathbf{I}_q - \mathbf{B}^\top\right)^{-1}\boldsymbol{\Lambda}^\top + \boldsymbol{\Theta}. \tag{1}$$

Let $\boldsymbol{\theta} \in \mathbb{R}^d$ collect the free parameters. Assume that $\boldsymbol{\Sigma}(\boldsymbol{\theta}) \in \mathbb{S}_{++}^p$. Denote the sample mean vector and covariance matrix by $\overline{\mathbf{y}} \in \mathbb{R}^p$ and $\mathbf{S} \in \mathbb{S}_+^p$, respectively, computed from a single group of independent and identically distributed observations $\mathbf{y}_i$, $i = 1, \ldots, n$, of $\mathbf{y}$, which is the setting of all estimation expressions below. Define

$$\mathbf{s} = \begin{pmatrix} \overline{\mathbf{y}} \\ \mathrm{vech}(\mathbf{S}) \end{pmatrix} \in \mathbb{R}^h, \qquad \boldsymbol{\sigma}(\boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{\mu}_y(\boldsymbol{\theta}) \\ \mathrm{vech}\left\{\boldsymbol{\Sigma}(\boldsymbol{\theta})\right\} \end{pmatrix} \in \mathbb{R}^h, \qquad h = p + \frac{p(p+1)}{2},$$

where vech stacks the lower triangular entries, including the diagonal. A common choice for estimating $\boldsymbol{\theta}$ jointly is ML, which requires further distributional assumptions (Browne, 1974; Shapiro, 2007; see Bollen, 2026, Chapter 4, pp. 201–203, Chapter 5, pp. 352–353, for an overview). JS, JSA, and MIIV proceed differently. They first estimate the factor loadings and latent regression slopes equation by equation, and then the remaining parameters conditional on these estimates.

2 Estimation of Factor Loadings and Latent Regression Slopes

We use a scaling indicator parameterization (Bollen, 1996, 2019; Burghgraeve et al., 2021), fixing one loading per latent variable to one and the corresponding intercept to zero (Bollen, 1989). Each scaling indicator is assumed to load only on its corresponding latent variable. The loadings and slopes are obtained from the sample covariances, with intercepts estimated subsequently. To facilitate the presentation of the

general expressions, we first present an example, based on the model in Figure 1, that illustrates how JS and MIIV address measurement error in a scaling indicator used as a predictor.

Example (A factor loading and a latent regression slope). Consider four indicators of η_1, with y_1 as its scaling indicator, so that $y_1 = \eta_1 + \epsilon_1$ and $y_r = \nu_r + \lambda_r \eta_1 + \epsilon_r$ for $r = 2, 3, 4$. Assume $\mathbb{E}(\epsilon_r) = 0$, $\mathrm{Cov}(\eta_1, \epsilon_r) = 0$, and $\mathrm{Cov}(\epsilon_r, \epsilon_s) = 0$ for all $r \neq s$. The objective in this example is to estimate λ_2. Writing $\psi = \mathrm{Var}(\eta_1)$ and $\omega_1 = \mathrm{Var}(\epsilon_1)$, the population slope from regressing y_2 directly on y_1 is

$$\frac{\mathrm{Cov}(y_1, y_2)}{\mathrm{Var}(y_1)} = \lambda_2 \frac{\psi}{\psi + \omega_1}.$$

Thus, measurement error in y_1 attenuates the slope. JS estimates the latent predictor from its scaling indicator. Under joint normality, it can be shown that

$$\mathbb{E}(\eta_1 \mid y_1) = \mu_1 + R(y_1 - \mu_1), \qquad R = \frac{\psi}{\psi + \omega_1},$$

where $\mu_1 = \mathbb{E}(\eta_1) = \mathbb{E}(y_1)$ (Burghgraeve et al., 2021). The other indicators provide information about ω_1, because $\mathrm{Cov}(y_1, y_3) = \lambda_3 \psi$, $\mathrm{Cov}(y_1, y_4) = \lambda_4 \psi$, and $\mathrm{Cov}(y_3, y_4) = \lambda_3 \lambda_4 \psi$. Consequently,

$$\psi = \frac{\mathrm{Cov}(y_1, y_3)\mathrm{Cov}(y_1, y_4)}{\mathrm{Cov}(y_3, y_4)}, \qquad \hat{\omega}_1 = s_{11} - \frac{s_{13} s_{14}}{s_{34}},$$

where s_{rs} denotes a sample covariance. This is the preliminary error variance estimate from one pair of other indicators. With the small sample correction used in JS, the estimated reliability adjustment and fitted latent predictor are

$$\hat{R} = \frac{s_{11} - c_n \hat{\omega}_1}{s_{11}}, \qquad c_n = \frac{n-3}{n-1}, \qquad \tilde{\eta}_1 = \bar{y}_1 + \hat{R}(y_1 - \bar{y}_1).$$

Regressing y_2 on $\tilde{\eta}_1$, with an intercept, gives

$$\hat{\lambda}_{2,\mathrm{JS}} = \frac{\hat{R}\, s_{12}}{\hat{R}^2 s_{11}} = \frac{s_{12}}{s_{11} - c_n \hat{\omega}_1}.$$

MIIV instead substitutes $\eta_1 = y_1 - \epsilon_1$ into the equation for y_2 (Bollen, 1996), giving $y_2 = \nu_2 + \lambda_2 y_1 + u$ with $u = \epsilon_2 - \lambda_2 \epsilon_1$. The explanatory variable y_1 correlates with the composite error, $\mathrm{Cov}(y_1, u) = -\lambda_2 \omega_1$, whereas y_3 satisfies $\mathrm{Cov}(y_3, u) = 0$. It can therefore serve as an instrument when $\mathrm{Cov}(y_3, y_1) \neq 0$, giving

$$\mathrm{Cov}(y_3, y_2) = \lambda_2\, \mathrm{Cov}(y_3, y_1), \qquad \hat{\lambda}_{2,\mathrm{MIIV}} = \frac{s_{32}}{s_{31}}.$$

With several instruments, such as y_3 and y_4, two-stage least squares first regresses y_1 on the instruments and then regresses y_2 on the fitted values, including intercepts in both regressions.

The same steps apply to a latent regression slope. In the model of Figure 1, which is also used in the simulation study below, $\eta_2 = \alpha_2 + B_{21}\eta_1 + \zeta_2$ and $y_5 = \eta_2 + \epsilon_5$. Substitution gives $y_5 = \alpha_2 + B_{21}\eta_1 + \zeta_2 + \epsilon_5$, which has the form of the equation for y_2, with $\zeta_2 + \epsilon_5$ in place of ϵ_2. Because ζ_2 and ϵ_5 are uncorrelated with η_1 and with $\epsilon_1, \ldots, \epsilon_4$, the same calculations as for λ_2 give, with y_5 in place of y_2,

$$\hat{B}_{21,\mathrm{JS}} = \frac{s_{15}}{s_{11} - c_n \hat{\omega}_1}, \qquad \hat{B}_{21,\mathrm{MIIV}} = \frac{s_{35}}{s_{31}}.$$

Moreover, for either estimator, $\hat{\nu}_2 = \bar{y}_2 - \hat{\lambda}_2 \bar{y}_1$ and $\hat{\alpha}_2 = \bar{y}_5 - \hat{B}_{21}\bar{y}_1$. Although not shown in this example, we also note JSA follows the JS calculation using a normalized aggregate of eligible indicators in place of y_1, as described below.

2.1 *James–Stein Estimation*

We now present the general expressions behind this example. As highlighted, JS and JSA proceed equation by equation. An equation belongs either to the measurement model, where it yields a factor loading, or to the structural model, where it yields a latent regression slope. The notation below refers to a single such equation. The expressions concern free loadings and slopes without equality constraints. All inverses and ratios are assumed well defined.

Scaling James–Stein Estimation

Let $\mathbf{f} \in \mathbb{R}^k$ collect the latent predictors of the equation to be estimated, and let $\mathbf{x} \in \mathbb{R}^k$ collect their scaling indicators in matching order. These are selected elements of $\boldsymbol{\eta}$ and $\mathbf{y}$, respectively. Because the scaling indicators have zero intercepts, unit loadings, and no cross loadings, their measurement equations reduce to

$$\mathbf{x} = \mathbf{f} + \mathbf{e},$$

where $\mathbf{e} \in \mathbb{R}^k$ contains the corresponding measurement errors. In the example, $f = \eta_1$, $x = y_1$, and $e = \epsilon_1$. Define

$$\boldsymbol{\mu}_x = \mathbb{E}(\mathbf{f}) = \mathbb{E}(\mathbf{x}) \in \mathbb{R}^k, \qquad \boldsymbol{\Sigma}_x = \mathrm{Var}(\mathbf{x}) \in \mathbb{S}^k_{++}, \qquad \boldsymbol{\Omega} = \mathrm{Var}(\mathbf{e}) \in \mathbb{S}^k_{+}.$$

Assume diagonal $\boldsymbol{\Omega}$ and $\mathrm{Cov}(\mathbf{f}, \mathbf{e}) = \mathbf{0}$. Under joint normality,

$$\mathbb{E}(\mathbf{f} \mid \mathbf{x}) = \boldsymbol{\mu}_x + \mathbf{R}(\mathbf{x} - \boldsymbol{\mu}_x), \qquad \mathbf{R} = (\boldsymbol{\Sigma}_x - \boldsymbol{\Omega})\boldsymbol{\Sigma}_x^{-1} \in \mathbb{R}^{k \times k}. \tag{2}$$

Otherwise, if normality does not hold, Equation (2) gives the best linear predictor (Burghgraeve et al., 2021). The preliminary error variance calculation illustrated in the example generalizes to

$$\hat{\omega}_j = s_{jj} - \frac{s_{js} s_{jt}}{s_{st}},$$

for three distinct indicators y_j, y_s, y_t loading only on the same latent variable and having mutually uncorrelated measurement errors. Estimates are averaged over all such pairs, with bounds applied to the corresponding estimated communalities. The resulting scaling indicator error variances form diagonal $\widehat{\boldsymbol{\Omega}} \in \mathbb{R}^{k \times k}$ and need not equal the final model estimates.

Using the sample covariance matrix $\mathbf{S}_x \in \mathbb{S}^k_{++}$ of the proxies $\mathbf{x}$, the estimated reliability adjustment and fitted predictors are

$$\hat{\mathbf{R}} = (\mathbf{S}_x - c_n \widehat{\boldsymbol{\Omega}})\mathbf{S}_x^{-1}, \qquad c_n = \frac{n-3}{n-1},$$

and $\tilde{\mathbf{f}} = \overline{\mathbf{x}} + \hat{\mathbf{R}}(\mathbf{x} - \overline{\mathbf{x}})$.

These fitted latent predictors are then used to estimate one measurement model or latent variable model equation. Let $\mathbf{a} \in \mathbb{R}^k$ contain the corresponding loadings or slopes. The response is the dependent indicator for a measurement model equation or the dependent latent variable's scaling indicator for a latent variable model equation. Let δ denote its error: ϵ_2 for response y_2 and $\zeta_2 + \epsilon_5$ for response y_5 in the example. Assume $\mathrm{Cov}(\mathbf{f}, \delta) = \mathbf{0}$ and $\mathrm{Cov}(\mathbf{e}, \delta) = \mathbf{0}$. Let $\mathbf{X} \in \mathbb{R}^{n \times k}$ contain the centered proxy observations, with rows $\left(\mathbf{x}_i - \overline{\mathbf{x}}\right)^\top$, and let $\mathbf{d} \in \mathbb{R}^n$ contain the centered response observations. The centered fitted predictor matrix is $\tilde{\mathbf{X}} = \mathbf{X}\hat{\mathbf{R}}^\top$, giving

$$\hat{\mathbf{a}}_{\mathrm{JS}} = \left(\tilde{\mathbf{X}}^\top \tilde{\mathbf{X}}\right)^{-1} \tilde{\mathbf{X}}^\top \mathbf{d} = \left(\mathbf{S}_x - c_n \widehat{\boldsymbol{\Omega}}\right)^{-1} \mathbf{s}_{xd}, \tag{3}$$

where $\mathbf{s}_{xd} \in \mathbb{R}^k$ contains the sample covariances between proxies and response.

Aggregated James–Stein Estimation

JSA replaces each scaling indicator by a normalized, reliability weighted aggregate (Burghgraeve et al., 2021). Eligible indicators load only on the corresponding latent predictor. Their measurement errors are mutually uncorrelated across all selected indicators and uncorrelated with δ, excluding the dependent indicator from its own predictor aggregate.

For each latent predictor f in $\mathbf{f}$, let $\mathbf{y}_f \in \mathbb{R}^r$ contain its eligible indicators, with intercepts $\boldsymbol{\nu}_f \in \mathbb{R}^r$, loadings $\boldsymbol{\lambda}_f \in \mathbb{R}^r$, and diagonal measurement error covariance matrix $\mathbf{Q}_f \in \mathbb{S}^r_+$. For weights $\mathbf{w} \in \mathbb{R}^r$,

$$g = \frac{\mathbf{w}^\top(\mathbf{y}_f - \boldsymbol{\nu}_f)}{\mathbf{w}^\top \boldsymbol{\lambda}_f} = f + e_g, \qquad \mathrm{Var}(e_g) = \frac{\mathbf{w}^\top \mathbf{Q}_f \mathbf{w}}{\left(\mathbf{w}^\top \boldsymbol{\lambda}_f\right)^2},$$

where e_g is the aggregate's measurement error. Using preliminary error variance estimates $\hat{\mathbf{Q}}_f$ and sample covariance matrix $\mathbf{S}_f \in \mathbb{S}^r_{++}$, the weights maximize estimated reliability:

$$\widehat{\mathbf{w}} \in \arg\max_{\mathbf{w}\neq\mathbf{0}}\left(1 - \frac{\mathbf{w}^\top \hat{\mathbf{Q}}_f \mathbf{w}}{\mathbf{w}^\top \mathbf{S}_f \mathbf{w}}\right).$$

With preliminary scaling JS loadings $\hat{\boldsymbol{\lambda}}_f$, the centered aggregate and its estimated error variance are

$$\hat{g} = \frac{\widehat{\mathbf{w}}^\top(\mathbf{y}_f - \overline{\mathbf{y}}_f)}{\widehat{\mathbf{w}}^\top \hat{\boldsymbol{\lambda}}_f}, \qquad \hat{\omega}_g = \frac{\widehat{\mathbf{w}}^\top \hat{\mathbf{Q}}_f \widehat{\mathbf{w}}}{\left(\widehat{\mathbf{w}}^\top \hat{\boldsymbol{\lambda}}_f\right)^2},$$

assuming a nonzero normalization denominator.

The centered aggregates replace the columns of $\mathbf{X}$, and their error variance estimates form the diagonal of $\widehat{\boldsymbol{\Omega}}$, in matching order. Recomputing $\mathbf{S}_x$ and $\mathbf{s}_{xd}$ from these aggregates and applying Equation (3) gives $\hat{\mathbf{a}}_{\text{JSA}}$, with the same response. Thus, JS and JSA share Equation (3) and differ in the construction of the proxies and their measurement error variances (Burghgraeve et al., 2021).

2.2 *Model-Implied Instrumental-Variable Estimation*

For MIIV, $\mathbf{x}$ and $\mathbf{X}$ again denote the scaling indicators and their centered observations. Substituting $\mathbf{f} = \mathbf{x} - \mathbf{e}$ produces the composite error $u = \delta - \mathbf{a}^\top \mathbf{e}$. After centering and stacking observations (Bollen, 1996), $\mathbf{d} = \mathbf{X}\mathbf{a} + \mathbf{u}$, where $\mathbf{u} \in \mathbb{R}^n$ contains the centered composite errors. The model identifies instruments $\mathbf{z} \in \mathbb{R}^\ell$ satisfying the validity and relevance conditions $\text{Cov}(\mathbf{z}, u) = \mathbf{0}$, $\text{rank}\{\text{Cov}(\mathbf{z}, \mathbf{x})\} = k$, and $\ell \geq k$ (Bollen, 1996; Bollen & Bauer, 2004). Let $\mathbf{Z} \in \mathbb{R}^{n\times\ell}$ contain their centered observations. Regressing the explanatory variables on the instruments gives

$$\hat{\mathbf{X}} = \mathbf{P}\mathbf{X}, \qquad \mathbf{P} = \mathbf{Z}\left(\mathbf{Z}^\top\mathbf{Z}\right)^{-1}\mathbf{Z}^\top \in \mathbb{R}^{n\times n}.$$

Regressing the response on these fitted values then gives

$$\hat{\mathbf{a}}_{\text{MIIV}} = \left(\hat{\mathbf{X}}^\top\hat{\mathbf{X}}\right)^{-1}\hat{\mathbf{X}}^\top\mathbf{d} = \left(\mathbf{X}^\top\mathbf{P}\mathbf{X}\right)^{-1}\mathbf{X}^\top\mathbf{P}\mathbf{d}.$$

Both approaches regress the response on fitted predictors. JS and JSA obtain them through an estimated reliability adjustment, whereas MIIV projects the scaling indicators onto model-implied instruments.

3 Estimation of Variance, Covariance, and Intercept Parameters

After estimating the factor loadings and latent regression slopes, the remaining (residual) variance and covariance parameters can be estimated using several least squares procedures (see Maydeu-Olivares et al., 2026, for a comparison of such procedures with MIIV). For JS, JSA, and MIIV, we use reweighted least squares, the default in the implementations considered, holding the estimated loadings and slopes fixed.

Substituting these estimates into Equation (1) gives

$$\boldsymbol{\Sigma}(\theta_2) = \widehat{\mathbf{H}}\boldsymbol{\Psi}(\theta_2)\widehat{\mathbf{H}}^\top + \boldsymbol{\Theta}(\theta_2), \qquad \widehat{\mathbf{H}} = \hat{\boldsymbol{\Lambda}}\left(\mathbf{I}_q - \hat{\mathbf{B}}\right)^{-1} \in \mathbb{R}^{p\times q}.$$

In this expression, $\theta_2 \in \mathbb{R}^{d_2}$, a subvector of θ, collects the free variance and covariance parameters in $\boldsymbol{\Theta}$ and $\boldsymbol{\Psi}$. With $\widehat{\mathbf{H}}$ held fixed, the implied covariance moments are linear in θ_2 (Browne, 1974; Ernst et al., 2023), that is, $\text{vech}\{\boldsymbol{\Sigma}(\theta_2)\} = \hat{\mathbf{c}} + \hat{\mathbf{G}}\theta_2$. Here, $t = p(p+1)/2$, $\hat{\mathbf{c}} \in \mathbb{R}^t$ contains the fixed component, and $\hat{\mathbf{G}} \in \mathbb{R}^{t\times d_2}$ maps the free parameters to the implied moments, incorporating any linear equality restrictions.

For a weight matrix $\widehat{\mathbf{W}} \in \mathbb{S}^t_{++}$, the weighted least squares estimator is

$$\hat{\theta}_2 = \left(\hat{\mathbf{G}}^\top\widehat{\mathbf{W}}\hat{\mathbf{G}}\right)^{-1}\hat{\mathbf{G}}^\top\widehat{\mathbf{W}}\{\text{vech}\,(\mathbf{S}) - \hat{\mathbf{c}}\}, \tag{4}$$

assuming full column rank of $\hat{\mathbf{G}}$. Reweighted least squares updates the normal theory weights using the current implied covariance matrix (Browne, 1974; Lee & Jennrich, 1979):

$$\widehat{\mathbf{W}}^{(\tau)} = \frac{1}{2}\mathbf{D}_p^\top\left[\left\{\hat{\boldsymbol{\Sigma}}^{(\tau)}\right\}^{-1} \otimes \left\{\hat{\boldsymbol{\Sigma}}^{(\tau)}\right\}^{-1}\right]\mathbf{D}_p, \qquad \hat{\boldsymbol{\Sigma}}^{(\tau)} = \boldsymbol{\Sigma}(\hat{\theta}_2^{(\tau)}).$$

Here, $\otimes$ denotes the Kronecker product, and $\mathbf{D}_p \in \mathbb{R}^{p^2 \times t}$ is the duplication matrix satisfying $\text{vec}(\mathbf{S}) = \mathbf{D}_p \, \text{vech}(\mathbf{S})$. Substituting $\widehat{\mathbf{W}}^{(\tau)}$ into Equation (4) gives $\hat{\theta}_2^{(\tau+1)}$. This procedure continues until convergence, assuming positive definite implied covariance matrices throughout. The resulting $\hat{\boldsymbol{\Theta}}$ and $\widehat{\boldsymbol{\Psi}}$ are not constrained to be positive semidefinite.

Finally, intercepts are estimated from the sample means. When all nonscaling indicator intercepts and structural intercepts are free, the mean structure is saturated. Let $\hat{\boldsymbol{\mu}}_\eta \in \mathbb{R}^q$ contain the sample means of the scaling indicators, ordered by latent variable. Then

$$\hat{\boldsymbol{\nu}} = \bar{\mathbf{y}} - \hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{\mu}}_\eta \in \mathbb{R}^p, \qquad \hat{\boldsymbol{\alpha}} = (\mathbf{I}_q - \hat{\mathbf{B}})\hat{\boldsymbol{\mu}}_\eta \in \mathbb{R}^q. \tag{5}$$

The expressions in Equation (5) preserve the fixed zero scaling indicator intercepts and reproduce the sample means. Additional mean structure restrictions are accommodated by generalized least squares, fitting $\boldsymbol{\mu}_y = \boldsymbol{\nu} + \widehat{\mathbf{H}}\boldsymbol{\alpha}$ to $\bar{\mathbf{y}}$ under those restrictions, with weight matrix $\hat{\boldsymbol{\Sigma}}^{-1}$ where $\hat{\boldsymbol{\Sigma}} = \boldsymbol{\Sigma}(\hat{\theta}_2)$.

Together, these stages estimate the free entries of $\boldsymbol{\Lambda}$, $\mathbf{B}$, $\boldsymbol{\Theta}$, $\boldsymbol{\Psi}$, $\boldsymbol{\nu}$, and $\boldsymbol{\alpha}$. Although loadings and slopes are held fixed during covariance estimation, their sampling uncertainty is included in the standard error calculations.

4 Analytical Standard Errors

Burghgraeve et al. (2021) derived a variance expression for latent regression slopes accounting for the estimated reliability adjustment and used bootstrap standard errors in their evaluations. Here, analytical standard errors follow from differentiating the complete estimation procedure with respect to the sample moments.[2] For $T \in \{\text{JS}, \text{JSA}\}$, write $\hat{\theta}_T = g_{n,T}(\mathbf{s})$, where $g_{n,T}$ maps the h sample moments to the d parameter estimates through the estimation stages described above.

Suppose

$$\sqrt{n}\{\mathbf{s} - \boldsymbol{\sigma}(\theta)\} \xrightarrow{d} N_h(\mathbf{0}, \boldsymbol{\Gamma}), \qquad \boldsymbol{\Gamma} \in \mathbb{S}_+^h,$$

where $\boldsymbol{\Gamma}$ is the joint asymptotic covariance matrix of the sample means and covariances. Under the required moment and differentiability conditions,

$$\hat{\mathbf{V}}_T = \frac{1}{n}\hat{\mathbf{J}}_T\hat{\boldsymbol{\Gamma}}\hat{\mathbf{J}}_T^\top \in \mathbb{S}_+^d, \qquad \hat{\mathbf{J}}_T = \frac{\partial g_{n,T}(\mathbf{s})}{\partial \mathbf{s}^\top}\Big|_{\mathbf{s}=\mathbf{s}_{\text{obs}}} \in \mathbb{R}^{d \times h}.$$

The Jacobian propagates uncertainty through all stages, including the preliminary error variances and intercept estimates. It is evaluated analytically by default, using implicit differentiation for the converged reweighted least squares equations. For JSA, it also includes derivatives of the aggregation weights and normalization loadings.

The matrix $\boldsymbol{\Gamma}$ can be estimated using normal theory or empirical moments through order four (Browne, 1984). Normal theory estimates may use the sample or fitted covariance matrix. For parameter $j = 1, \ldots, d$, $\widehat{\text{SE}}(\hat{\theta}_{T,j}) = \sqrt{[\hat{\mathbf{V}}_T]_{jj}} \in [0, \infty)$.

5 Simulation Design and Performance Evaluation

In this section we describe the design of the simulation study and the measures used to evaluate the standard errors.

5.1 *Population Model and Estimands*

We study the model in Figure 1, with three latent variables and four indicators each. The latent variable model is

[2] Standard errors for MIIV are not treated here. They are well documented, with two-stage least squares and delta method expressions for the loadings and slopes (Bollen, 1996; Fisher & Bollen, 2020; see also Maydeu-Olivares et al., 2019), and with expressions for the subsequently estimated variance and covariance parameters that account for uncertainty in the first-stage estimates (Bollen & Maydeu-Olivares, 2007; Jin et al., 2021).

$$\begin{aligned}\eta_2 &= \alpha_2 + B_{21}\eta_1 + \zeta_2,\\ \eta_3 &= \alpha_3 + B_{31}\eta_1 + B_{32}\eta_2 + \zeta_3.\end{aligned} \tag{6}$$

The design is partly based on previous studies. The measurement models follow the simulations of Burghgraeve et al. (2021), with four indicators per latent variable and a common reliability of .6 or .8 for all indicators. The structural model follows their empirical illustration, which has the same two structural equations. Moreover, Jin et al. (2021) used a recursive latent variable model with several equations and standardized slopes between 0.1 and 0.6, which the present model follows partly with $B_{21} = 0.6$, $B_{31} = 0.3$, and $B_{32} = 0.5$. The disturbance variances were chosen so that all latent variables have unit variance. With $\text{Var}(\eta_1) = 1$, setting $\text{Var}(\zeta_2) = 0.64$ and $\text{Var}(\zeta_3) = 0.48$ gives $\text{Var}(\eta_2) = \text{Var}(\eta_3) = 1$. The slopes therefore equal their standardized values. The implied correlations are .60 between η_1 and η_2, .60 between η_1 and η_3, and .68 between η_2 and η_3. The scaling indicators have loadings fixed to one and the remaining loadings range from 0.8 to 1.2. The mean structure has $\mathbb{E}(\eta_1) = 1$, $\alpha_2 = \alpha_3 = 0.5$, and $\nu_j = 0.2$ for the nonscaling indicators, where j indexes the indicators in Figure 1. The main estimands are all free factor loadings, latent regression slopes, measurement error variances, and latent variances and disturbance variances.

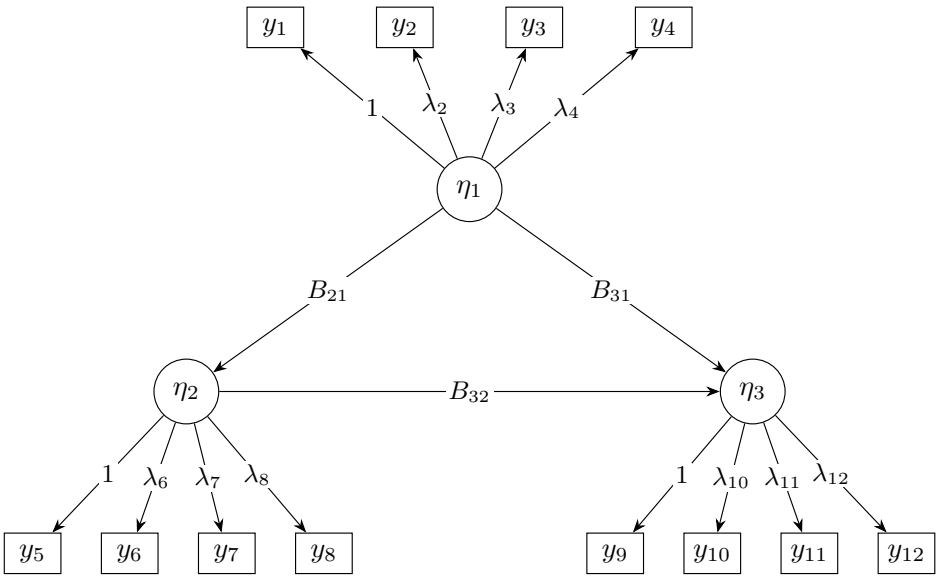

Figure 1 Diagram of the simulation model with three latent variables and their indicators.

Note. Scaling indicator loadings are fixed to one. The population latent regression slopes are $B_{21} = 0.6$, $B_{31} = 0.3$, and $B_{32} = 0.5$. Intercepts, measurement errors, disturbances, and variances are omitted for clarity.

5.2 *Simulation Conditions and Estimation Approaches*

We employed a fully factorial design with two factors: sample size ($n \in \{100, 200, 300, 400, 500, 600\}$) and indicator reliability ($\rho \in \{.6, .8\}$). This yielded $6 \times 2 = 12$ conditions, each replicated 1,500 times. The sample sizes cover the small to moderate range in which the equation-by-equation estimators are expected to differ most from ML (e.g., Burghgraeve et al., 2021; Dhaene & Rosseel, 2024; Jin et al., 2021). The reliability levels match the low and high conditions of Burghgraeve et al. (2021), as mentioned before, and determine the size of the reliability adjustment in JS and JSA.

With respect to the estimation approaches, JS, JSA, MIIV, and ML were applied as implemented in lavaan with the default options. In particular, $\boldsymbol{\Gamma}$ was estimated under normal theory with the fitted covariance matrix for all parameters in JS and JSA and for the (residual) variance, covariance, and intercept parameters in MIIV. For MIIV, each equation used all instruments implied by the model (Bollen & Bauer, 2004). For ML, the standard errors were based on the expected Fisher information matrix.

5.3 *Data Generation*

Data were generated from the population model with all stochastic components normal and mutually independent. The exogenous latent variable was generated as $\eta_1 \sim \mathcal{N}(1, 1)$ and the structural disturbances as $\zeta_2 \sim \mathcal{N}(0, 0.64)$ and $\zeta_3 \sim \mathcal{N}(0, 0.48)$, from which η_2 and η_3 follow by Equation (6). The measurement error of each indicator was generated as $\epsilon_j \sim \mathcal{N}\{0, \lambda_j^2(1-\rho)/\rho\}$, where λ_j is the loading of indicator j. This entails that every indicator has reliability $\rho \in \{.6, .8\}$ in its condition. The indicators were then obtained from the measurement equations $y_j = \nu_j + \lambda_j \eta_l + \epsilon_j$, where η_l is the latent variable measured by indicator j in Figure 1, with $\nu_j = 0$ and $\lambda_j = 1$ for the scaling indicators. The observed variables are therefore multivariate normal with the mean vector and covariance matrix implied by the population values.

5.4 *Convergence, Outliers, and Inadmissible Solutions*

Replications were screened in two steps, and a replication that failed either step for any estimator was removed for all four. A solution was nonconverged when any estimate, standard error, or confidence interval bound of an estimand was missing or non-finite. Among the converged replications, outliers were point estimates more than three interquartile ranges below the first or above the third quartile, per condition, estimator, and estimand. Inadmissible solutions, with a nonpositive estimate of a measurement error, latent, or disturbance variance, were retained and counted.

5.5 *Performance Measures*

Performance was assessed with the measures implemented in the simhelpers package (Joshi & Pustejovsky, 2025), computed for every estimand. These measures, with their empirical and Monte Carlo standard error (MCSE) calculations, are listed in (**tbl-performance?**). Jackknife MCSEs are used for the empirical standard deviation (SD) of the estimates and for the ratio of the mean squared standard error (SE^2) to the empirical variance (VAR). The MCSE of coverage is available in closed form. The SE/SD ratio and the SE^2/VAR ratio compare the same quantities on the standard deviation and on the variance scale. Only the latter, for which an MCSE is available, is shown in the figures. Coverage between .925 and .975 and SE^2/VAR ratios between 0.90 and 1.10 were considered acceptable.

We also note that, although the evaluation is focused on the standard errors, the relative bias of the estimates was also computed, as standard errors of biased estimates are of limited use for inference. It is reported in the text, but not in the figures. For complete tables of all measures and their MCSEs, we refer the reader to Appendix B.

Performance measure	Empirical calculation	MCSE
Relative bias	$(\bar{\theta} - \theta)/\theta$	$S_{\hat{\theta}}/(\sqrt{K}\,\lvert\theta\rvert)$
SD	$S_{\hat{\theta}} = \sqrt{\frac{1}{K-1}\sum_{k=1}^{K}\left(\hat{\theta}_k - \bar{\theta}\right)^2}$	$\sqrt{\frac{K-1}{K}\sum_{j=1}^{K}\left(S_{(j)} - S_{\hat{\theta}}\right)^2}$
SE/SD ratio	$\overline{SE}/S_{\hat{\theta}}$	—
SE^2/VAR ratio	$\nu = \overline{SE^2}/S^2_{\hat{\theta}}$	$\sqrt{\frac{K-1}{K}\sum_{j=1}^{K}\left(\nu_{(j)} - \nu\right)^2}$
Coverage	$c = \frac{1}{K}\sum_{k=1}^{K} I(L_k \le \theta \le U_k)$	$\sqrt{c(1-c)/K}$

Note. $\hat{\theta}_k$ is the estimate of parameter θ in replication k ($k = 1, \ldots, K$) and $\bar{\theta}$ the mean of the K estimates. $\widehat{SE}_k$ is the estimated standard error in replication k, $\overline{SE} = \frac{1}{K}\sum_k \widehat{SE}_k$, $\overline{SE^2} = \frac{1}{K}\sum_k \widehat{SE}_k^2$, and L_k, U_k are the confidence interval limits. The subscript (j) marks a leave-one-out jackknife replicate. The dash marks an MCSE that is not computed.

6 Results

6.1 *Convergence, Outliers, and Inadmissible Solutions*

All four estimators converged in every replication of every condition. Outlier screening removed 30 of the 18,000 replications (0.17%), concentrated at $n = 100$ with reliability .6, where 0.8% of the replications were removed, flagged most often through JS (0.7% of the replications in that condition). In every other condition at most 0.3% of the replications were removed.[3] No inadmissible solutions occurred.

6.2 *Relative Bias*

JS, JSA, and ML were essentially unbiased, with absolute relative bias of at most 4.4% for any parameter and condition. MIIV was also nearly unbiased except at reliability .6 and small samples. At $n = 100$ its loadings were slightly underestimated, by 6.3 to 6.9%, whereas its latent and disturbance variances were overestimated by 11.0 to 12.4%. From $n = 300$ onward, all estimators were within 4.6% of the population values.

6.3 *Standard Error Performance*

Each figure presents reliability .6 in the upper half and .8 in the lower half. Figure 2 shows the latent regression slopes, Figure 3 one loading per latent variable, Figure 4 the measurement error variances of the scaling indicators and one nonscaling indicator per latent variable, and Figure 5 the variance of η_1 and the two disturbance variances. Omitted loadings and measurement error variances behave similarly to those shown for the same latent variable, with SE^2/VAR ratios that differ by less than 0.18 and coverage by .03 in every condition.

Overall, standard error performance was acceptable. Across the 1,296 combinations of 27 estimands, 12 conditions, and 4 estimators, 96.4% of coverage rates fell between .925 and .975 and 98.2% of SE^2/VAR ratios between 0.90 and 1.10. Coverage ranged from .88 to .98 and the ratios from 0.91 to 1.24. All 23 ratios outside the acceptable range exceeded 1.10, and of the 47 coverage rates outside it only 1 exceeded .975. Of these, 15 ratios and 43 coverage rates occurred at $n = 100$ or $n = 200$. From $n = 300$ onward, coverage ranged from .92 to .96 and the ratios from 0.91 to 1.11. Three patterns account for most of these departures.

First, JS standard errors for B_{31} and B_{32} exceed the acceptable range at $n = 100$ and reliability .6, with SE^2/VAR ratios of 1.24 and 1.18. Coverage nonetheless exceeds .975 only for B_{31} at $n = 100$. Its SD is also the largest, 0.23 versus 0.13–0.17. Both slopes belong to the η_3 equation, the only equation with two latent predictors, and the departure is confined to the lowest reliability and largest at the smallest sample size. JSA, which aggregates indicators within each latent variable, shows only a slight excess for B_{31} at $n = 100$, with a ratio of 1.12.

Second, MIIV loading intervals fall below the acceptable coverage range at reliability .6, with coverage of .88–.90 at $n = 100$, compared with .93–.96 for the other estimators, despite SE^2/VAR ratios of 0.97–1.14 and the smallest SD. This discrepancy reflects the downward loading bias reported above, consistent with the known small-sample bias of IV estimators (Jin et al., 2021), which they found largest when all model-implied instruments are used, as here, and diminishes slowly as sample size increases.

Third, at $n = 100$ and reliability .6, MIIV overestimates the standard errors of the latent and disturbance variances, with ratios of 1.11–1.19, but its coverage rates of .96–.97 remain within the acceptable range. ML, in contrast, has ratios close to 1 but coverage rates of .91–.93 for the disturbance variances. JS and JSA fall between them. At reliability .8 and $n = 100$, the coverage rate for the disturbance variance of η_2 ranges from .915 to .922 for ML, JS, and JSA, just below the acceptable range. Measurement error variances show less contrast, with coverage rates from .91 to .96 across estimators and a common dip at $n = 100$.

[3] As a sensitivity check, all performance measures were recomputed without the outlier screening, retaining all 1,500 replications of every condition. The conclusions did not change. Appendix A reports, for each condition, the largest difference in each performance measure with and without screening, and the complete unscreened measures are available in the project repository.

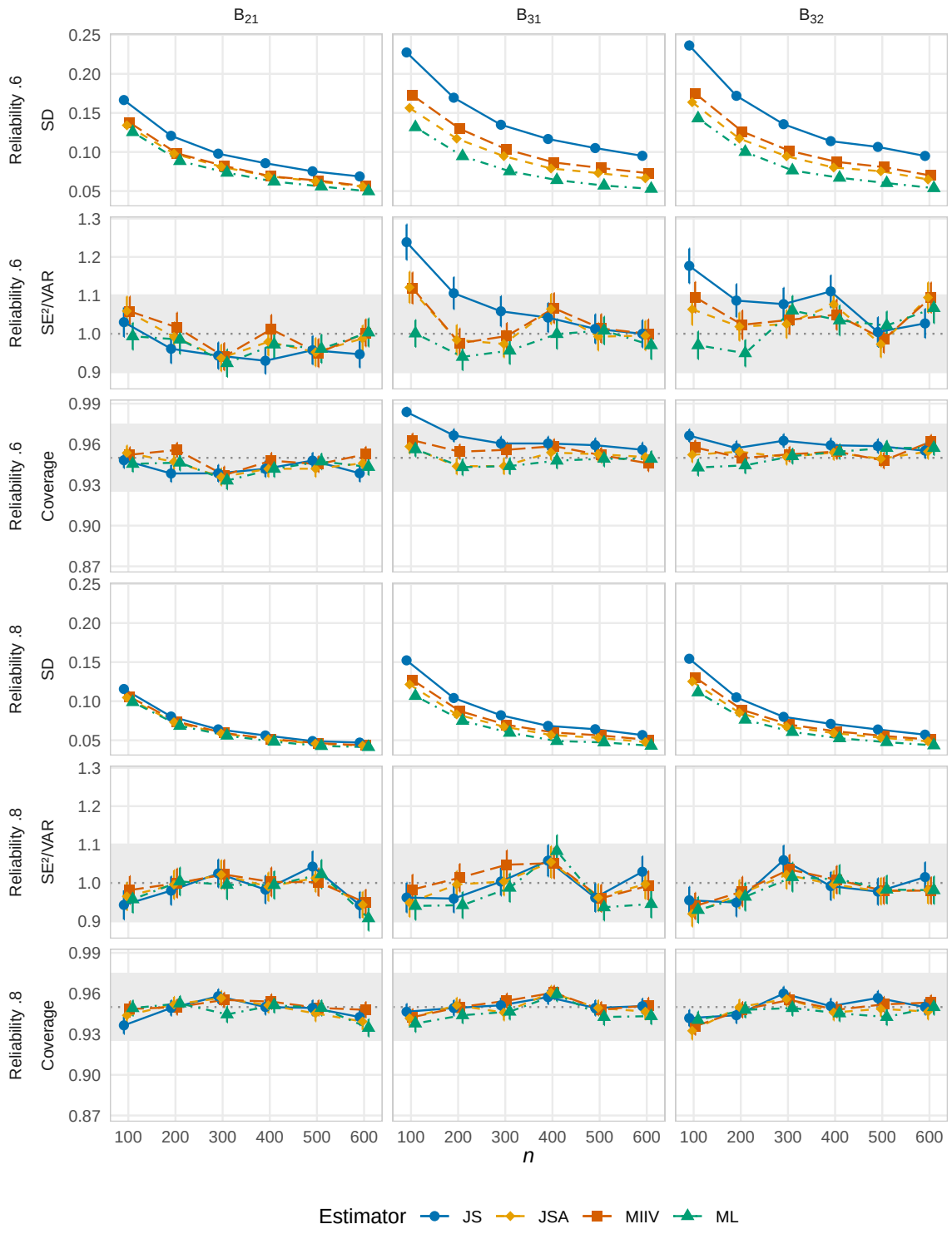

Figure 2 Performance of the Standard Errors of the Latent Regression Slopes

Note. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood. Dotted lines mark the reference values 1 and .95, and shaded bands the acceptable ranges of 0.90 to 1.10 and .925 to .975. Error bars represent Monte Carlo standard errors.

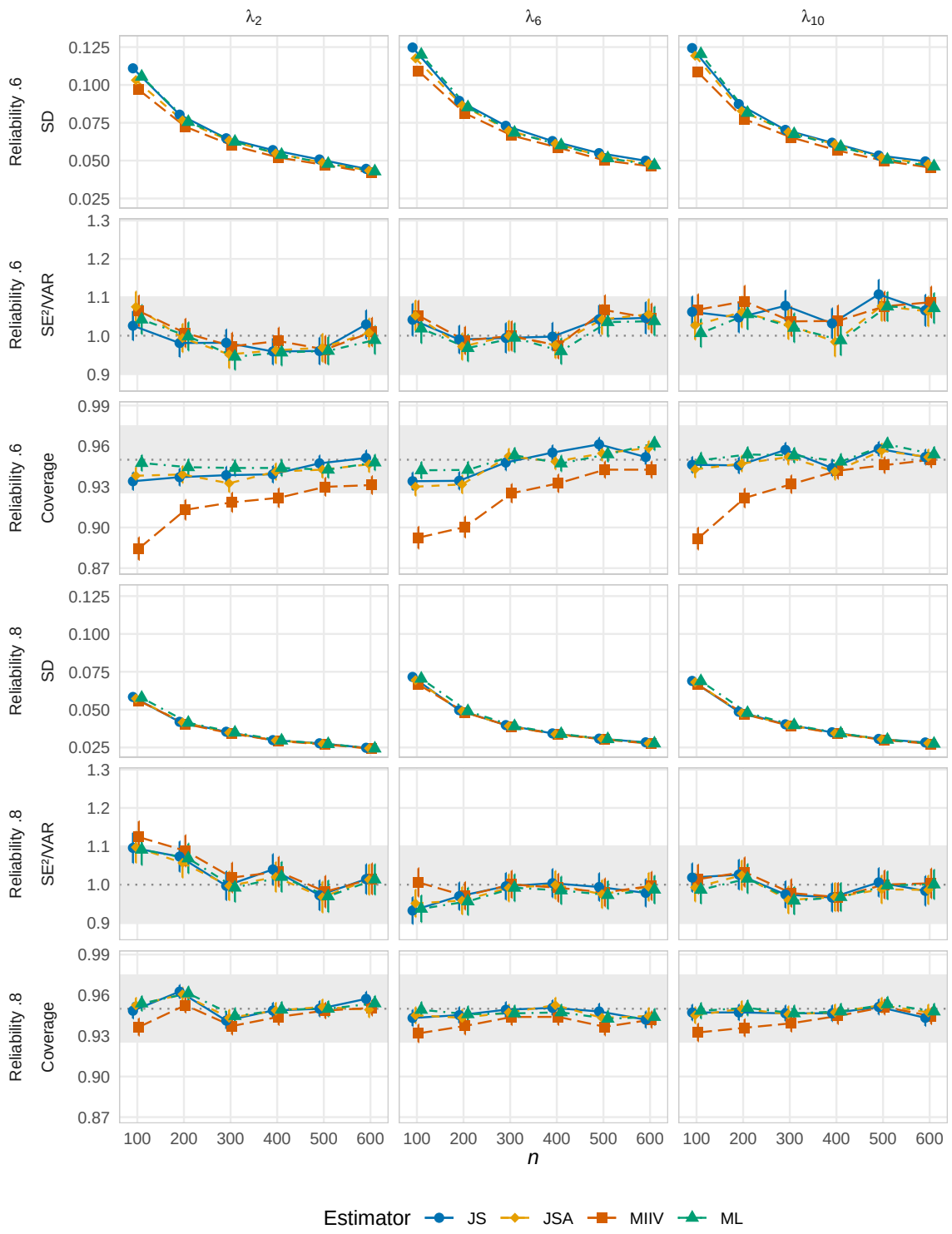

Figure 3 Performance of the Standard Errors of One Factor Loading per Latent Variable

Note. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood. Dotted lines mark the reference values 1 and .95, and shaded bands the acceptable ranges of 0.90 to 1.10 and .925 to .975. Error bars represent Monte Carlo standard errors.

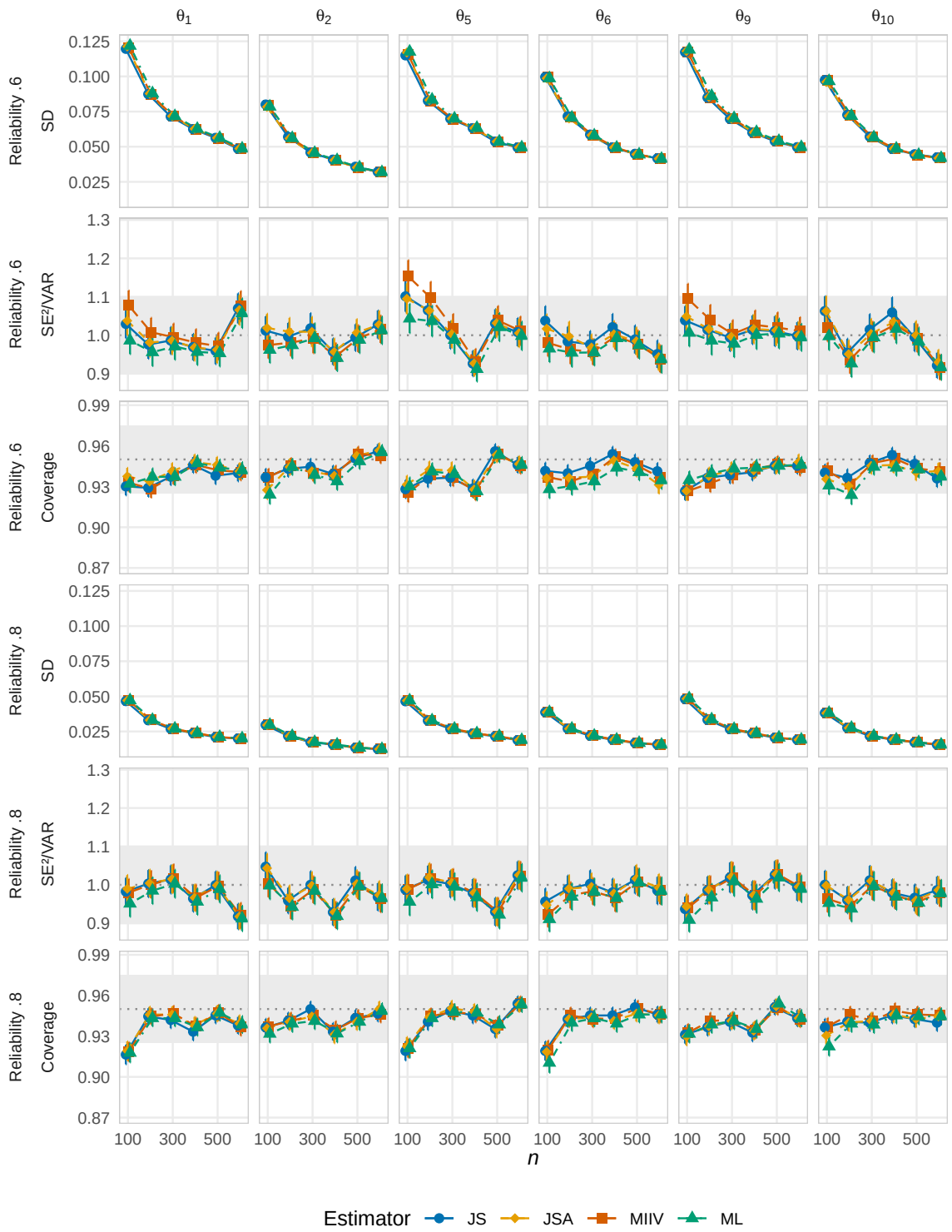

Figure 4 Performance of the Standard Errors of the Measurement Error Variances of the Scaling Indicator and One Nonscaling Indicator per Latent Variable

Note. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood. Dotted lines mark the reference values 1 and .95, and shaded bands the acceptable ranges of 0.90 to 1.10 and .925 to .975. Error bars represent Monte Carlo standard errors.

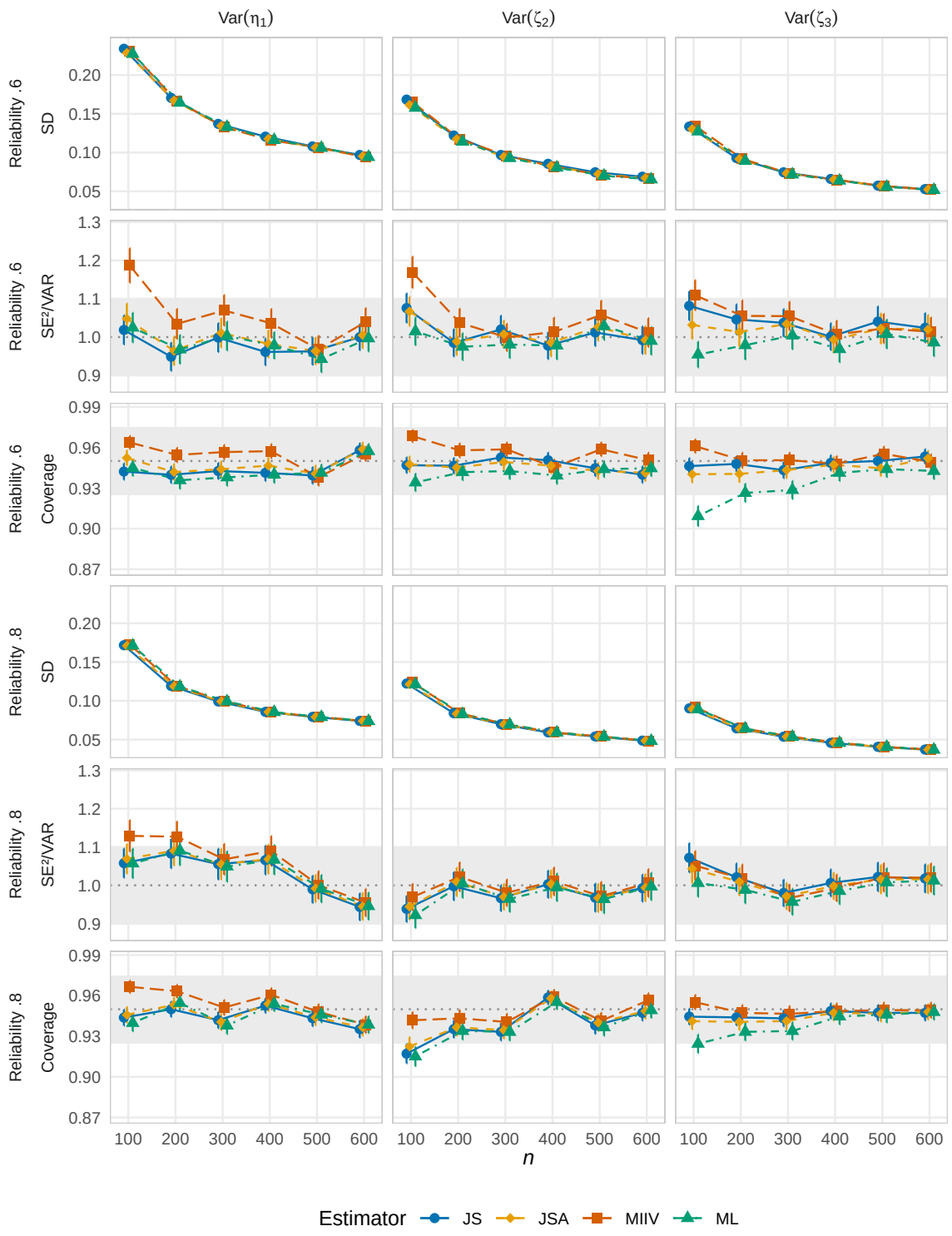

Figure 5 Performance of the Standard Errors of the Latent and Disturbance Variances

Note. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood. Dotted lines mark the reference values 1 and .95, and shaded bands the acceptable ranges of 0.90 to 1.10 and .925 to .975. Error bars represent Monte Carlo standard errors.

7 Discussion

This article presented JS and JSA estimation as implemented in the R package lavaan, starting from a worked example and using MIIV as a point of comparison using a common notation. It also developed and presented analytical standard errors for JS and JSA by applying the delta method to the complete estimation procedure. A reproducible simulation study then evaluated their accuracy against MIIV and ML under correct model specification.

The main finding is that, for a correctly specified recursive model with normally distributed data, the analytical standard errors of JS and JSA performed comparably to those of MIIV and ML. Both estimators were essentially unbiased in all conditions, and their coverage rates and SE^2/VAR ratios were acceptable from $n = 300$ onward for essentially every parameter. The largest departure concerned JS at reliability .6 and $n = 100$, where the standard errors of the slopes in the η_3 equation were too large. This departure was confined to the equation with two latent predictors. JSA, which aggregates indicators within each latent variable, was much less affected. In small samples with low reliability, JSA is therefore preferable to JS. These results extend those of Burghgraeve et al. (2021), who found JS less efficient than JSA for latent regression estimation and JSA competitive with ML. The empirical standard deviations observed here show the same ranking, JS above JSA above ML. Their bootstrap standard errors for JS were too small in small samples, however, whereas our analytical standard errors were too large. Whether this difference reflects the standard error methods or differences in the simulation conditions requires further investigation.

The MIIV results provide the contrast and broadly align with earlier studies. Bollen (2019), summarizing Bollen et al. (2007), reports few differences between MIIV and ML under correct specification with normally distributed data. Our results largely agree in terms of bias and standard error accuracy from $n = 300$ onward, although MIIV remained less precise than ML for the latent regression slopes. Jin et al. (2021) found that low scaling indicator reliability can lead to biased MIIV estimates in small samples and, under that condition, to the least accurate standard errors for the variance parameters. Related patterns appeared here at reliability .6 and $n = 100$, with loading bias and undercoverage, alongside inflated standard errors for latent and disturbance variances. JS and JSA showed neither the loading bias nor the undercoverage in these conditions.

With this article and its simulation, however, we did not intend to compare the approaches exhaustively. Given that JS and JSA have received little attention so far, our aim was to show what they share with MIIV and that both can perform comparably to ML. We hope this encourages further evaluation of JS and JSA across a broader range of conditions, including departures from normality and misspecification of the measurement and latent variable models, where their equation-by-equation structure may matter most.

Data and Code Availability

All analyses were conducted in R 4.6.0 (R Core Team, 2026) using the development version 0.7.3.3364 of the lavaan package (Rosseel, 2012) for estimation, installed from the GitHub repository at commit ef620c8, simhelpers 0.3.1 (Joshi & Pustejovsky, 2025) for performance evaluation, and ggplot2 4.0.3 (Wickham, 2016) for the figures. The computational environment was pinned with Nix (Dolstra et al., 2004) via the development version 0.18.6 of the rix package (Rodrigues & Baumann, 2026), installed from the GitHub repository at commit a08ee5c, and a Dockerfile that builds the same environment inside a container is also provided. The article was written with the apaquarto extension (Schneider, 2026). The simulation used a single seed with the default random number generator in R, the Mersenne Twister. We also highlight that all conditions and replications were run sequentially in a fixed order from this single seed, without parallel processing. Thus, running the simulation within the pinned environment reproduces the stored results, as long as the sequential execution is not changed. All code, simulation results, and the manuscript source, together with a description of all files and instructions to reproduce the simulation study and the manuscript, are archived on Zenodo at https://doi.org/10.5281/zenodo.23730879.

Funding

This project was funded by the Special Research Fund (BOF) of Ghent University (BOF/24J/2023/144).

Conflicts of Interest

The authors report there are no competing interests to declare.

7.4 *Use of Generative AI*

Generative AI (Claude Opus 5.5, Anthropic) was used to create all figures, with the TikZ and R code based on earlier files written by the first author, and to improve the flow of the Introduction, Results, and Discussion. Commits containing AI generated code or text are marked as such in the commit history of the project repository. All AI output and suggestions were checked by the first author, and the authors take full responsibility for the content of this article.

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Appendix A: Sensitivity to Outlier Screening

(**tbl-sensitivity?**) reports, for each condition with removed replications, the number removed and the largest absolute change in each performance measure when they are retained.

n	Reliability	Removed	Relative bias	SD	SE^2/VAR	Coverage
100	.6	12	0.002	0.004	0.044	0.003
200	.6	5	0.001	0.002	0.034	0.002
300	.6	2	0.001	0.001	0.019	0.001
400	.6	2	0.001	0.001	0.019	0.001
600	.6	1	0.001	0.001	0.022	0.001
	.8	4	0.001	0.001	0.013	0.001
200	.8	1	0.000	0.000	0.015	0.001
300	.8	2	0.000	0.000	0.017	0.001
600	.8	1	0.000	0.000	0.015	0.001

Note. Removed is the number of replications out of 1,500 removed by the outlier screening. The remaining columns give, for each performance measure, the largest absolute difference across the four estimators and all estimands between the reported value and the value with the removed replications retained. Conditions without removed replications are omitted.

Appendix B: Complete Performance Measures

(**tbl-rb-6?**) to (**tbl-cov-8?**) report, for each performance measure and reliability, the values for every estimator, sample size, and estimand shown in the figures, with the MCSE in parentheses.

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
B_{21}	JS	−0.8 (0.7)	0.2 (0.5)	−0.6 (0.4)	−0.2 (0.4)	0.2 (0.3)	−0.2 (0.3)
	JSA	−2.0 (0.6)	−0.2 (0.4)	−1.1 (0.3)	−0.7 (0.3)	0.1 (0.3)	−0.3 (0.2)
	MIIV	−1.3 (0.6)	0.0 (0.4)	−1.0 (0.4)	−0.5 (0.3)	0.2 (0.3)	−0.2 (0.2)
	ML	0.3 (0.5)	0.8 (0.4)	−0.4 (0.3)	0.0 (0.3)	0.5 (0.2)	0.0 (0.2)
B_{31}	JS	2.6 (2.0)	0.0 (1.5)	−0.4 (1.2)	−0.1 (1.0)	0.1 (0.9)	−1.9 (0.8)
	JSA	0.9 (1.4)	−0.8 (1.0)	0.4 (0.8)	−0.2 (0.7)	1.1 (0.6)	−1.3 (0.6)
	MIIV	1.3 (1.5)	−1.1 (1.1)	0.6 (0.9)	0.3 (0.7)	1.2 (0.7)	−1.1 (0.6)
	ML	0.3 (1.1)	−1.0 (0.8)	0.3 (0.6)	0.0 (0.6)	0.5 (0.5)	0.0 (0.5)
B_{32}	JS	−1.1 (1.2)	0.2 (0.9)	0.5 (0.7)	0.5 (0.6)	0.3 (0.5)	0.8 (0.5)
	JSA	−1.3 (0.8)	0.0 (0.6)	−0.8 (0.5)	0.0 (0.4)	−0.8 (0.4)	0.5 (0.3)
	MIIV	−3.0 (0.9)	−0.6 (0.7)	−1.5 (0.5)	−0.5 (0.5)	−1.2 (0.4)	0.3 (0.4)
	ML	1.5 (0.7)	1.5 (0.5)	0.2 (0.4)	0.6 (0.3)	0.0 (0.3)	0.3 (0.3)
λ_2	JS	−0.9 (0.4)	−0.3 (0.3)	−0.6 (0.2)	−0.3 (0.2)	−0.3 (0.2)	−0.4 (0.1)
	JSA	−1.2 (0.3)	−0.3 (0.2)	−0.6 (0.2)	−0.3 (0.2)	−0.3 (0.2)	−0.3 (0.1)
	MIIV	−6.9 (0.3)	−3.4 (0.2)	−2.6 (0.2)	−1.8 (0.2)	−1.4 (0.2)	−1.3 (0.1)
	ML	0.4 (0.3)	0.4 (0.2)	−0.1 (0.2)	0.1 (0.2)	0.1 (0.2)	0.0 (0.1)
λ_6	JS	−0.8 (0.4)	−0.5 (0.3)	0.1 (0.2)	−0.1 (0.2)	0.0 (0.2)	−0.1 (0.1)
	JSA	−1.0 (0.3)	−0.4 (0.2)	0.0 (0.2)	−0.1 (0.2)	0.0 (0.2)	0.0 (0.1)
	MIIV	−6.6 (0.3)	−3.4 (0.2)	−2.0 (0.2)	−1.5 (0.2)	−1.2 (0.1)	−1.0 (0.1)
	ML	0.8 (0.3)	0.4 (0.2)	0.5 (0.2)	0.3 (0.2)	0.2 (0.1)	0.2 (0.1)
λ_{10}	JS	−0.6 (0.4)	−0.7 (0.3)	−0.2 (0.2)	−0.1 (0.2)	−0.1 (0.2)	−0.2 (0.1)
	JSA	−0.6 (0.3)	−0.8 (0.2)	−0.1 (0.2)	0.0 (0.2)	−0.1 (0.1)	−0.2 (0.1)
	MIIV	−6.3 (0.3)	−3.6 (0.2)	−2.1 (0.2)	−1.5 (0.2)	−1.3 (0.1)	−1.1 (0.1)
	ML	1.1 (0.3)	0.1 (0.2)	0.4 (0.2)	0.4 (0.2)	0.2 (0.1)	0.1 (0.1)
θ_1	JS	−4.0 (0.5)	−2.4 (0.3)	−1.5 (0.3)	−1.0 (0.2)	−0.6 (0.2)	−1.1 (0.2)
	JSA	−2.7 (0.5)	−1.7 (0.3)	−1.1 (0.3)	−0.7 (0.2)	−0.3 (0.2)	−0.8 (0.2)
	MIIV	−4.3 (0.5)	−2.6 (0.3)	−1.7 (0.3)	−1.1 (0.2)	−0.7 (0.2)	−1.1 (0.2)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	ML	−2.3 (0.5)	−1.5 (0.3)	−0.9 (0.3)	−0.6 (0.2)	−0.2 (0.2)	−0.7 (0.2)
θ_2	JS	−1.5 (0.5)	0.0 (0.3)	0.3 (0.3)	−0.3 (0.2)	−0.2 (0.2)	−0.2 (0.2)
	JSA	−2.6 (0.5)	−0.6 (0.3)	−0.1 (0.3)	−0.6 (0.2)	−0.5 (0.2)	−0.5 (0.2)
	MIIV	−0.3 (0.5)	0.5 (0.3)	0.6 (0.3)	−0.1 (0.2)	−0.1 (0.2)	−0.1 (0.2)
	ML	−2.8 (0.5)	−0.7 (0.3)	−0.2 (0.3)	−0.7 (0.2)	−0.5 (0.2)	−0.5 (0.2)
θ_5	JS	−3.5 (0.4)	−2.4 (0.3)	−1.6 (0.3)	−1.2 (0.2)	−0.7 (0.2)	−0.8 (0.2)
	JSA	−2.4 (0.5)	−1.8 (0.3)	−1.3 (0.3)	−0.9 (0.2)	−0.5 (0.2)	−0.7 (0.2)
	MIIV	−3.9 (0.4)	−2.6 (0.3)	−1.8 (0.3)	−1.3 (0.2)	−0.9 (0.2)	−0.9 (0.2)
	ML	−1.5 (0.5)	−1.4 (0.3)	−1.0 (0.3)	−0.7 (0.2)	−0.4 (0.2)	−0.5 (0.2)
θ_6	JS	−0.8 (0.5)	−0.3 (0.3)	−0.1 (0.3)	−0.1 (0.2)	−0.2 (0.2)	−0.4 (0.2)
	JSA	−1.8 (0.5)	−0.9 (0.3)	−0.4 (0.3)	−0.4 (0.2)	−0.5 (0.2)	−0.6 (0.2)
	MIIV	0.4 (0.5)	0.2 (0.3)	0.3 (0.3)	0.1 (0.2)	0.0 (0.2)	−0.3 (0.2)
	ML	−2.1 (0.5)	−1.0 (0.3)	−0.5 (0.3)	−0.4 (0.2)	−0.5 (0.2)	−0.7 (0.2)
θ_9	JS	−4.4 (0.5)	−2.0 (0.3)	−1.9 (0.3)	−1.3 (0.2)	−0.9 (0.2)	−0.6 (0.2)
	JSA	−3.8 (0.5)	−1.7 (0.3)	−1.7 (0.3)	−1.1 (0.2)	−0.7 (0.2)	−0.5 (0.2)
	MIIV	−5.1 (0.5)	−2.4 (0.3)	−2.3 (0.3)	−1.5 (0.2)	−1.1 (0.2)	−0.8 (0.2)
	ML	−2.3 (0.5)	−0.9 (0.3)	−1.2 (0.3)	−0.7 (0.2)	−0.4 (0.2)	−0.2 (0.2)
θ_{10}	JS	−1.6 (0.5)	−0.2 (0.3)	−0.6 (0.3)	−0.3 (0.2)	−0.4 (0.2)	−0.2 (0.2)
	JSA	−2.1 (0.5)	−0.5 (0.3)	−0.8 (0.3)	−0.5 (0.2)	−0.5 (0.2)	−0.3 (0.2)
	MIIV	0.1 (0.5)	0.6 (0.3)	−0.1 (0.3)	0.0 (0.2)	−0.1 (0.2)	0.0 (0.2)
	ML	−2.6 (0.5)	−0.7 (0.3)	−0.9 (0.3)	−0.6 (0.2)	−0.6 (0.2)	−0.4 (0.2)
$\mathrm{Var}(\eta_1)$	JS	1.4 (0.6)	−0.2 (0.4)	0.8 (0.4)	0.2 (0.3)	0.4 (0.3)	0.8 (0.3)
	JSA	2.6 (0.6)	0.4 (0.4)	1.3 (0.3)	0.5 (0.3)	0.6 (0.3)	1.0 (0.2)
	MIIV	12.4 (0.6)	5.4 (0.4)	4.6 (0.3)	3.0 (0.3)	2.6 (0.3)	2.6 (0.2)
	ML	0.5 (0.6)	−0.7 (0.4)	0.5 (0.3)	−0.1 (0.3)	0.1 (0.3)	0.5 (0.2)
$\mathrm{Var}(\zeta_2)$	JS	0.7 (0.7)	0.4 (0.5)	−0.1 (0.4)	0.1 (0.3)	−0.5 (0.3)	−0.1 (0.3)
	JSA	2.2 (0.7)	1.2 (0.5)	0.6 (0.4)	0.5 (0.3)	−0.2 (0.3)	0.2 (0.3)
	MIIV	11.0 (0.7)	5.6 (0.5)	3.7 (0.4)	2.7 (0.3)	1.7 (0.3)	1.7 (0.3)
	ML	−1.3 (0.6)	−0.5 (0.5)	−0.5 (0.4)	−0.4 (0.3)	−0.7 (0.3)	−0.4 (0.3)
$\mathrm{Var}(\zeta_3)$	JS	3.4 (0.7)	1.4 (0.5)	0.1 (0.4)	1.1 (0.4)	0.0 (0.3)	0.6 (0.3)
	JSA	2.2 (0.7)	0.7 (0.5)	−0.3 (0.4)	0.6 (0.4)	−0.2 (0.3)	0.3 (0.3)
	MIIV	11.1 (0.7)	5.2 (0.5)	2.8 (0.4)	2.9 (0.3)	1.7 (0.3)	1.8 (0.3)
	ML	−2.9 (0.7)	−1.9 (0.5)	−1.9 (0.4)	−0.6 (0.3)	−1.2 (0.3)	−0.5 (0.3)

Note. Monte Carlo standard errors in parentheses. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood.

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
B_{21}	JS	0.4 (0.5)	0.4 (0.3)	0.4 (0.3)	−0.2 (0.2)	−0.2 (0.2)	0.1 (0.2)
	JSA	−0.1 (0.5)	0.2 (0.3)	0.3 (0.3)	−0.2 (0.2)	−0.2 (0.2)	0.1 (0.2)
	MIIV	0.0 (0.5)	0.3 (0.3)	0.3 (0.3)	−0.1 (0.2)	−0.1 (0.2)	0.2 (0.2)
	ML	0.6 (0.4)	0.4 (0.3)	0.6 (0.2)	0.0 (0.2)	0.0 (0.2)	0.2 (0.2)
B_{31}	JS	−1.8 (1.3)	−3.2 (0.9)	−0.3 (0.7)	−0.2 (0.6)	0.0 (0.6)	−0.3 (0.5)
	JSA	−0.9 (1.0)	−2.0 (0.7)	−0.1 (0.6)	0.0 (0.5)	0.0 (0.5)	−0.3 (0.4)
	MIIV	−0.4 (1.1)	−2.1 (0.8)	0.1 (0.6)	−0.1 (0.5)	0.0 (0.5)	−0.5 (0.4)
	ML	−0.6 (0.9)	−1.4 (0.6)	−0.5 (0.5)	0.1 (0.4)	0.0 (0.4)	−0.1 (0.4)
B_{32}	JS	0.6 (0.8)	1.3 (0.5)	0.1 (0.4)	0.1 (0.4)	0.0 (0.3)	0.2 (0.3)
	JSA	−0.4 (0.6)	0.4 (0.4)	−0.2 (0.3)	−0.2 (0.3)	0.0 (0.3)	0.0 (0.3)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	MIIV	−1.2 (0.7)	0.1 (0.5)	−0.4 (0.4)	−0.3 (0.3)	0.0 (0.3)	0.0 (0.3)
	ML	0.5 (0.6)	0.4 (0.4)	0.4 (0.3)	−0.1 (0.3)	0.1 (0.2)	0.0 (0.2)
λ_2	JS	−0.3 (0.2)	0.1 (0.1)	0.1 (0.1)	−0.2 (0.1)	0.0 (0.1)	0.0 (0.1)
	JSA	−0.3 (0.2)	0.1 (0.1)	0.1 (0.1)	−0.2 (0.1)	0.0 (0.1)	0.0 (0.1)
	MIIV	−2.2 (0.2)	−0.9 (0.1)	−0.6 (0.1)	−0.6 (0.1)	−0.4 (0.1)	−0.3 (0.1)
	ML	0.2 (0.2)	0.4 (0.1)	0.2 (0.1)	0.0 (0.1)	0.1 (0.1)	0.1 (0.1)
λ_6	JS	−0.2 (0.2)	−0.1 (0.1)	−0.1 (0.1)	−0.1 (0.1)	0.0 (0.1)	0.0 (0.1)
	JSA	−0.1 (0.2)	−0.1 (0.1)	−0.1 (0.1)	−0.1 (0.1)	0.0 (0.1)	0.0 (0.1)
	MIIV	−2.1 (0.2)	−1.1 (0.1)	−0.7 (0.1)	−0.6 (0.1)	−0.4 (0.1)	−0.4 (0.1)
	ML	0.4 (0.2)	0.2 (0.1)	0.1 (0.1)	0.0 (0.1)	0.1 (0.1)	0.1 (0.1)
λ_{10}	JS	0.0 (0.2)	−0.2 (0.1)	−0.1 (0.1)	−0.1 (0.1)	−0.3 (0.1)	−0.1 (0.1)
	JSA	−0.1 (0.2)	−0.2 (0.1)	−0.1 (0.1)	−0.1 (0.1)	−0.3 (0.1)	−0.1 (0.1)
	MIIV	−2.1 (0.2)	−1.2 (0.1)	−0.7 (0.1)	−0.6 (0.1)	−0.7 (0.1)	−0.4 (0.1)
	ML	0.5 (0.2)	0.1 (0.1)	0.1 (0.1)	0.0 (0.1)	−0.2 (0.1)	0.0 (0.1)
θ_1	JS	−4.0 (0.5)	−1.7 (0.3)	−0.9 (0.3)	−1.2 (0.2)	−0.6 (0.2)	−0.6 (0.2)
	JSA	−3.1 (0.5)	−1.2 (0.3)	−0.6 (0.3)	−1.0 (0.2)	−0.5 (0.2)	−0.4 (0.2)
	MIIV	−3.2 (0.5)	−1.4 (0.3)	−0.6 (0.3)	−1.1 (0.2)	−0.5 (0.2)	−0.5 (0.2)
	ML	−3.0 (0.5)	−1.1 (0.3)	−0.5 (0.3)	−1.0 (0.2)	−0.4 (0.2)	−0.4 (0.2)
θ_2	JS	−1.3 (0.5)	−1.0 (0.4)	−0.8 (0.3)	−0.5 (0.3)	−0.4 (0.2)	−0.4 (0.2)
	JSA	−1.9 (0.5)	−1.2 (0.4)	−1.0 (0.3)	−0.6 (0.3)	−0.5 (0.2)	−0.5 (0.2)
	MIIV	−0.3 (0.5)	−0.5 (0.4)	−0.5 (0.3)	−0.2 (0.3)	−0.2 (0.2)	−0.2 (0.2)
	ML	−1.9 (0.5)	−1.3 (0.4)	−1.0 (0.3)	−0.6 (0.3)	−0.5 (0.2)	−0.5 (0.2)
θ_5	JS	−3.0 (0.5)	−1.3 (0.3)	−0.9 (0.3)	−0.8 (0.2)	−0.9 (0.2)	−0.4 (0.2)
	JSA	−2.2 (0.5)	−0.9 (0.3)	−0.7 (0.3)	−0.6 (0.2)	−0.7 (0.2)	−0.3 (0.2)
	MIIV	−2.4 (0.5)	−1.0 (0.3)	−0.7 (0.3)	−0.7 (0.2)	−0.8 (0.2)	−0.3 (0.2)
	ML	−1.9 (0.5)	−0.8 (0.3)	−0.5 (0.3)	−0.5 (0.2)	−0.7 (0.2)	−0.2 (0.2)
θ_6	JS	−2.7 (0.5)	−0.9 (0.3)	−0.6 (0.3)	−0.6 (0.2)	−0.5 (0.2)	−0.3 (0.2)
	JSA	−3.2 (0.5)	−1.1 (0.3)	−0.7 (0.3)	−0.7 (0.2)	−0.6 (0.2)	−0.3 (0.2)
	MIIV	−1.7 (0.5)	−0.4 (0.3)	−0.2 (0.3)	−0.4 (0.2)	−0.3 (0.2)	−0.1 (0.2)
	ML	−3.3 (0.5)	−1.2 (0.3)	−0.8 (0.3)	−0.8 (0.2)	−0.6 (0.2)	−0.4 (0.2)
θ_9	JS	−2.2 (0.5)	−0.9 (0.3)	−1.0 (0.3)	−1.0 (0.2)	−0.8 (0.2)	−0.4 (0.2)
	JSA	−1.7 (0.5)	−0.6 (0.3)	−0.9 (0.3)	−0.9 (0.2)	−0.7 (0.2)	−0.3 (0.2)
	MIIV	−1.9 (0.5)	−0.7 (0.3)	−0.9 (0.3)	−1.0 (0.2)	−0.7 (0.2)	−0.4 (0.2)
	ML	−1.0 (0.5)	−0.3 (0.3)	−0.6 (0.3)	−0.7 (0.2)	−0.5 (0.2)	−0.2 (0.2)
θ_{10}	JS	−1.7 (0.5)	−0.2 (0.4)	−0.5 (0.3)	0.1 (0.2)	−0.3 (0.2)	−0.2 (0.2)
	JSA	−2.0 (0.5)	−0.3 (0.4)	−0.6 (0.3)	0.0 (0.2)	−0.3 (0.2)	−0.3 (0.2)
	MIIV	−0.5 (0.5)	0.4 (0.4)	−0.1 (0.3)	0.4 (0.2)	−0.1 (0.2)	0.0 (0.2)
	ML	−2.2 (0.5)	−0.4 (0.4)	−0.7 (0.3)	0.0 (0.2)	−0.4 (0.2)	−0.3 (0.2)
$\mathrm{Var}(\eta_1)$	JS	0.2 (0.4)	−0.8 (0.3)	−0.1 (0.3)	0.5 (0.2)	−0.1 (0.2)	−0.2 (0.2)
	JSA	0.4 (0.4)	−0.7 (0.3)	0.0 (0.3)	0.5 (0.2)	−0.1 (0.2)	−0.2 (0.2)
	MIIV	4.4 (0.4)	1.3 (0.3)	1.3 (0.3)	1.4 (0.2)	0.7 (0.2)	0.5 (0.2)
	ML	−0.3 (0.4)	−1.1 (0.3)	−0.3 (0.3)	0.3 (0.2)	−0.3 (0.2)	−0.3 (0.2)
$\mathrm{Var}(\zeta_2)$	JS	−1.6 (0.5)	−0.7 (0.3)	−0.2 (0.3)	−0.3 (0.2)	0.0 (0.2)	−0.1 (0.2)
	JSA	−1.5 (0.5)	−0.6 (0.3)	−0.1 (0.3)	−0.3 (0.2)	0.0 (0.2)	−0.1 (0.2)
	MIIV	2.4 (0.5)	1.3 (0.3)	1.1 (0.3)	0.7 (0.2)	0.7 (0.2)	0.5 (0.2)
	ML	−2.6 (0.5)	−1.1 (0.3)	−0.5 (0.3)	−0.5 (0.2)	−0.3 (0.2)	−0.3 (0.2)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
$\mathrm{Var}(\zeta_3)$	JS	−0.9 (0.5)	−0.7 (0.3)	−0.1 (0.3)	−0.1 (0.2)	0.0 (0.2)	0.0 (0.2)
	JSA	−1.7 (0.5)	−1.1 (0.3)	−0.4 (0.3)	−0.3 (0.2)	−0.2 (0.2)	−0.1 (0.2)
	MIIV	2.4 (0.5)	0.9 (0.3)	0.9 (0.3)	0.7 (0.2)	0.6 (0.2)	0.6 (0.2)
	ML	−3.3 (0.5)	−1.9 (0.3)	−0.9 (0.3)	−0.7 (0.2)	−0.5 (0.2)	−0.4 (0.2)

Note. Monte Carlo standard errors in parentheses. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood.

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
B_{21}	JS	0.1665 (0.0033)	0.1207 (0.0024)	0.0978 (0.0018)	0.0856 (0.0016)	0.0753 (0.0015)	0.0687 (0.0013)
	JSA	0.1342 (0.0025)	0.0978 (0.0019)	0.0812 (0.0015)	0.0690 (0.0013)	0.0625 (0.0012)	0.0558 (0.0010)
	MIIV	0.1374 (0.0026)	0.0982 (0.0019)	0.0822 (0.0015)	0.0688 (0.0013)	0.0634 (0.0012)	0.0561 (0.0010)
	ML	0.1254 (0.0023)	0.0884 (0.0017)	0.0734 (0.0014)	0.0622 (0.0012)	0.0559 (0.0011)	0.0497 (0.0009)
B_{31}	JS	0.2274 (0.0048)	0.1695 (0.0035)	0.1349 (0.0026)	0.1166 (0.0023)	0.1050 (0.0020)	0.0951 (0.0017)
	JSA	0.1563 (0.0029)	0.1171 (0.0024)	0.0947 (0.0017)	0.0789 (0.0015)	0.0730 (0.0013)	0.0663 (0.0013)
	MIIV	0.1725 (0.0033)	0.1298 (0.0026)	0.1032 (0.0018)	0.0866 (0.0016)	0.0795 (0.0015)	0.0729 (0.0014)
	ML	0.1317 (0.0024)	0.0946 (0.0018)	0.0753 (0.0014)	0.0641 (0.0012)	0.0570 (0.0010)	0.0528 (0.0010)
B_{32}	JS	0.2363 (0.0052)	0.1719 (0.0038)	0.1356 (0.0029)	0.1139 (0.0023)	0.1065 (0.0021)	0.0949 (0.0018)
	JSA	0.1637 (0.0033)	0.1171 (0.0021)	0.0945 (0.0017)	0.0801 (0.0015)	0.0754 (0.0013)	0.0648 (0.0012)
	MIIV	0.1750 (0.0034)	0.1262 (0.0024)	0.1016 (0.0019)	0.0873 (0.0017)	0.0806 (0.0014)	0.0698 (0.0013)
	ML	0.1430 (0.0028)	0.1001 (0.0019)	0.0764 (0.0014)	0.0670 (0.0013)	0.0602 (0.0011)	0.0537 (0.0010)
λ_2	JS	0.1110 (0.0022)	0.0804 (0.0016)	0.0648 (0.0012)	0.0570 (0.0010)	0.0508 (0.0009)	0.0446 (0.0008)
	JSA	0.1031 (0.0020)	0.0766 (0.0015)	0.0633 (0.0012)	0.0548 (0.0010)	0.0487 (0.0009)	0.0434 (0.0008)
	MIIV	0.0968 (0.0019)	0.0724 (0.0014)	0.0601 (0.0011)	0.0520 (0.0009)	0.0471 (0.0009)	0.0419 (0.0008)
	ML	0.1055 (0.0020)	0.0755 (0.0014)	0.0625 (0.0011)	0.0538 (0.0010)	0.0479 (0.0009)	0.0429 (0.0008)
λ_6	JS	0.1247 (0.0027)	0.0895 (0.0017)	0.0730 (0.0015)	0.0629 (0.0012)	0.0549 (0.0010)	0.0499 (0.0010)
	JSA	0.1175 (0.0023)	0.0866 (0.0016)	0.0699 (0.0014)	0.0612 (0.0011)	0.0529 (0.0010)	0.0479 (0.0009)
	MIIV	0.1092 (0.0021)	0.0813 (0.0015)	0.0666 (0.0013)	0.0586 (0.0010)	0.0501 (0.0009)	0.0462 (0.0009)
	ML	0.1199 (0.0024)	0.0852 (0.0016)	0.0684 (0.0013)	0.0601 (0.0011)	0.0516 (0.0009)	0.0469 (0.0009)
λ_{10}	JS	0.1243 (0.0025)	0.0873 (0.0017)	0.0703 (0.0013)	0.0618 (0.0012)	0.0534 (0.0009)	0.0495 (0.0010)
	JSA	0.1193 (0.0023)	0.0828 (0.0016)	0.0689 (0.0013)	0.0608 (0.0012)	0.0520 (0.0009)	0.0477 (0.0009)
	MIIV	0.1084 (0.0021)	0.0773 (0.0015)	0.0653 (0.0012)	0.0566 (0.0011)	0.0499 (0.0009)	0.0453 (0.0008)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	ML	0.1204	0.0816	0.0675	0.0591	0.0506	0.0462
		(0.0023)	(0.0016)	(0.0013)	(0.0012)	(0.0009)	(0.0009)
θ_1	JS	0.1196	0.0874	0.0715	0.0625	0.0563	0.0486
		(0.0022)	(0.0017)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
	JSA	0.1213	0.0878	0.0717	0.0627	0.0563	0.0487
		(0.0023)	(0.0017)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
	MIIV	0.1206	0.0870	0.0715	0.0623	0.0560	0.0485
		(0.0022)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
	ML	0.1218	0.0880	0.0717	0.0627	0.0562	0.0486
		(0.0023)	(0.0017)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
θ_2	JS	0.0800	0.0570	0.0459	0.0408	0.0358	0.0321
		(0.0015)	(0.0010)	(0.0009)	(0.0007)	(0.0006)	(0.0006)
	JSA	0.0783	0.0559	0.0456	0.0404	0.0352	0.0318
		(0.0015)	(0.0010)	(0.0009)	(0.0007)	(0.0006)	(0.0006)
	MIIV	0.0792	0.0562	0.0457	0.0404	0.0353	0.0318
		(0.0015)	(0.0010)	(0.0009)	(0.0007)	(0.0006)	(0.0006)
	ML	0.0785	0.0561	0.0455	0.0404	0.0352	0.0318
		(0.0014)	(0.0010)	(0.0009)	(0.0007)	(0.0006)	(0.0006)
θ_5	JS	0.1150	0.0828	0.0699	0.0632	0.0537	0.0495
		(0.0022)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
	JSA	0.1170	0.0832	0.0700	0.0632	0.0536	0.0495
		(0.0022)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
	MIIV	0.1154	0.0822	0.0696	0.0631	0.0534	0.0493
		(0.0022)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
	ML	0.1176	0.0835	0.0700	0.0633	0.0536	0.0495
		(0.0022)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
θ_6	JS	0.0996	0.0717	0.0586	0.0496	0.0449	0.0417
		(0.0019)	(0.0014)	(0.0010)	(0.0008)	(0.0008)	(0.0008)
	JSA	0.0987	0.0701	0.0582	0.0495	0.0446	0.0415
		(0.0019)	(0.0014)	(0.0010)	(0.0009)	(0.0008)	(0.0008)
	MIIV	0.0996	0.0709	0.0582	0.0494	0.0445	0.0414
		(0.0019)	(0.0014)	(0.0010)	(0.0009)	(0.0008)	(0.0008)
	ML	0.0987	0.0706	0.0579	0.0493	0.0444	0.0413
		(0.0019)	(0.0014)	(0.0010)	(0.0008)	(0.0008)	(0.0008)
θ_9	JS	0.1173	0.0849	0.0698	0.0601	0.0539	0.0497
		(0.0022)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
	JSA	0.1180	0.0854	0.0700	0.0602	0.0540	0.0497
		(0.0022)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
	MIIV	0.1172	0.0848	0.0699	0.0599	0.0538	0.0494
		(0.0022)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
	ML	0.1190	0.0861	0.0703	0.0604	0.0540	0.0497
		(0.0022)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0009)
θ_{10}	JS	0.0974	0.0726	0.0571	0.0484	0.0446	0.0424
		(0.0018)	(0.0014)	(0.0011)	(0.0009)	(0.0008)	(0.0008)
	JSA	0.0960	0.0719	0.0569	0.0486	0.0441	0.0419
		(0.0018)	(0.0014)	(0.0011)	(0.0009)	(0.0008)	(0.0007)
	MIIV	0.0972	0.0721	0.0568	0.0485	0.0441	0.0420
		(0.0018)	(0.0014)	(0.0011)	(0.0009)	(0.0008)	(0.0007)
	ML	0.0967	0.0719	0.0566	0.0486	0.0442	0.0419
		(0.0018)	(0.0014)	(0.0011)	(0.0009)	(0.0008)	(0.0007)
$\mathrm{Var}(\eta_1)$	JS	0.2339	0.1706	0.1370	0.1206	0.1080	0.0969
		(0.0045)	(0.0034)	(0.0026)	(0.0022)	(0.0020)	(0.0017)
	JSA	0.2286	0.1672	0.1345	0.1178	0.1065	0.0953
		(0.0045)	(0.0033)	(0.0026)	(0.0022)	(0.0020)	(0.0016)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	MIIV	0.2310 (0.0045)	0.1664 (0.0032)	0.1328 (0.0025)	0.1157 (0.0021)	0.1068 (0.0020)	0.0939 (0.0016)
	ML	0.2273 (0.0045)	0.1642 (0.0032)	0.1330 (0.0025)	0.1162 (0.0021)	0.1060 (0.0020)	0.0943 (0.0016)
$\mathrm{Var}(\zeta_2)$	JS	0.1683 (0.0032)	0.1221 (0.0023)	0.0969 (0.0018)	0.0855 (0.0016)	0.0746 (0.0013)	0.0689 (0.0013)
	JSA	0.1615 (0.0030)	0.1175 (0.0022)	0.0946 (0.0017)	0.0829 (0.0015)	0.0723 (0.0013)	0.0672 (0.0012)
	MIIV	0.1650 (0.0030)	0.1174 (0.0021)	0.0960 (0.0017)	0.0819 (0.0015)	0.0711 (0.0013)	0.0663 (0.0012)
	ML	0.1578 (0.0030)	0.1141 (0.0021)	0.0929 (0.0017)	0.0806 (0.0015)	0.0701 (0.0012)	0.0654 (0.0012)
$\mathrm{Var}(\zeta_3)$	JS	0.1335 (0.0025)	0.0930 (0.0018)	0.0746 (0.0013)	0.0660 (0.0012)	0.0573 (0.0011)	0.0529 (0.0010)
	JSA	0.1303 (0.0024)	0.0918 (0.0018)	0.0733 (0.0013)	0.0651 (0.0012)	0.0571 (0.0011)	0.0524 (0.0010)
	MIIV	0.1344 (0.0025)	0.0919 (0.0018)	0.0731 (0.0013)	0.0647 (0.0012)	0.0568 (0.0011)	0.0521 (0.0010)
	ML	0.1272 (0.0024)	0.0893 (0.0018)	0.0717 (0.0013)	0.0638 (0.0012)	0.0557 (0.0011)	0.0517 (0.0010)

Note. Monte Carlo standard errors in parentheses. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood.

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
B_{21}	JS	0.1156 (0.0024)	0.0804 (0.0015)	0.0639 (0.0011)	0.0561 (0.0010)	0.0489 (0.0009)	0.0470 (0.0008)
	JSA	0.1047 (0.0020)	0.0731 (0.0014)	0.0587 (0.0011)	0.0514 (0.0010)	0.0456 (0.0008)	0.0432 (0.0008)
	MIIV	0.1053 (0.0020)	0.0738 (0.0014)	0.0593 (0.0011)	0.0515 (0.0010)	0.0463 (0.0008)	0.0434 (0.0008)
	ML	0.0988 (0.0018)	0.0684 (0.0012)	0.0559 (0.0011)	0.0482 (0.0009)	0.0426 (0.0008)	0.0413 (0.0007)
B_{31}	JS	0.1522 (0.0032)	0.1040 (0.0020)	0.0820 (0.0015)	0.0683 (0.0013)	0.0641 (0.0012)	0.0567 (0.0011)
	JSA	0.1214 (0.0024)	0.0831 (0.0015)	0.0674 (0.0012)	0.0567 (0.0011)	0.0531 (0.0010)	0.0477 (0.0009)
	MIIV	0.1271 (0.0026)	0.0874 (0.0016)	0.0697 (0.0012)	0.0599 (0.0011)	0.0561 (0.0010)	0.0505 (0.0010)
	ML	0.1067 (0.0021)	0.0748 (0.0014)	0.0596 (0.0011)	0.0490 (0.0009)	0.0472 (0.0009)	0.0429 (0.0008)
B_{32}	JS	0.1543 (0.0030)	0.1048 (0.0021)	0.0799 (0.0014)	0.0710 (0.0013)	0.0638 (0.0011)	0.0572 (0.0011)
	JSA	0.1252 (0.0023)	0.0851 (0.0016)	0.0676 (0.0012)	0.0592 (0.0011)	0.0533 (0.0009)	0.0486 (0.0009)
	MIIV	0.1304 (0.0024)	0.0888 (0.0018)	0.0700 (0.0013)	0.0613 (0.0011)	0.0555 (0.0010)	0.0507 (0.0009)
	ML	0.1113 (0.0020)	0.0763 (0.0014)	0.0606 (0.0011)	0.0525 (0.0010)	0.0476 (0.0009)	0.0434 (0.0008)
λ_2	JS	0.0584 (0.0011)	0.0420 (0.0008)	0.0354 (0.0007)	0.0299 (0.0006)	0.0278 (0.0006)	0.0248 (0.0005)
	JSA	0.0573 (0.0011)	0.0416 (0.0008)	0.0348 (0.0007)	0.0297 (0.0006)	0.0274 (0.0006)	0.0244 (0.0005)
	MIIV	0.0559 (0.0010)	0.0405 (0.0008)	0.0341 (0.0007)	0.0292 (0.0006)	0.0270 (0.0006)	0.0242 (0.0005)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	ML	0.0578	0.0413	0.0348	0.0296	0.0272	0.0243
		(0.0011)	(0.0008)	(0.0007)	(0.0006)	(0.0006)	(0.0005)
λ_6	JS	0.0716	0.0496	0.0398	0.0343	0.0309	0.0284
		(0.0014)	(0.0009)	(0.0007)	(0.0006)	(0.0006)	(0.0005)
	JSA	0.0695	0.0490	0.0392	0.0338	0.0306	0.0277
		(0.0013)	(0.0009)	(0.0007)	(0.0006)	(0.0006)	(0.0005)
	MIIV	0.0666	0.0481	0.0386	0.0336	0.0303	0.0274
		(0.0012)	(0.0009)	(0.0007)	(0.0006)	(0.0006)	(0.0005)
	ML	0.0704	0.0490	0.0390	0.0339	0.0305	0.0276
		(0.0013)	(0.0009)	(0.0007)	(0.0006)	(0.0006)	(0.0005)
λ_{10}	JS	0.0690	0.0486	0.0403	0.0351	0.0307	0.0283
		(0.0013)	(0.0009)	(0.0007)	(0.0007)	(0.0006)	(0.0006)
	JSA	0.0681	0.0477	0.0398	0.0344	0.0304	0.0278
		(0.0013)	(0.0009)	(0.0007)	(0.0007)	(0.0006)	(0.0005)
	MIIV	0.0664	0.0469	0.0391	0.0341	0.0299	0.0273
		(0.0013)	(0.0009)	(0.0007)	(0.0007)	(0.0006)	(0.0005)
	ML	0.0688	0.0478	0.0397	0.0342	0.0301	0.0274
		(0.0013)	(0.0009)	(0.0007)	(0.0007)	(0.0006)	(0.0005)
θ_1	JS	0.0466	0.0329	0.0269	0.0238	0.0210	0.0200
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0004)
	JSA	0.0471	0.0331	0.0270	0.0239	0.0211	0.0201
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0004)
	MIIV	0.0472	0.0332	0.0270	0.0239	0.0211	0.0200
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0004)
	ML	0.0471	0.0332	0.0270	0.0239	0.0211	0.0200
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0004)
θ_2	JS	0.0297	0.0218	0.0174	0.0157	0.0134	0.0125
		(0.0005)	(0.0004)	(0.0003)	(0.0003)	(0.0002)	(0.0002)
	JSA	0.0296	0.0217	0.0174	0.0156	0.0135	0.0125
		(0.0005)	(0.0004)	(0.0003)	(0.0003)	(0.0002)	(0.0002)
	MIIV	0.0299	0.0218	0.0174	0.0157	0.0135	0.0125
		(0.0005)	(0.0004)	(0.0003)	(0.0003)	(0.0002)	(0.0002)
	ML	0.0297	0.0217	0.0174	0.0156	0.0135	0.0125
		(0.0005)	(0.0004)	(0.0003)	(0.0003)	(0.0002)	(0.0002)
θ_5	JS	0.0465	0.0327	0.0269	0.0236	0.0216	0.0189
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0003)
	JSA	0.0470	0.0328	0.0270	0.0236	0.0216	0.0189
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0003)
	MIIV	0.0469	0.0328	0.0269	0.0236	0.0216	0.0189
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0003)
	ML	0.0471	0.0328	0.0270	0.0237	0.0217	0.0189
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0003)
θ_6	JS	0.0387	0.0270	0.0219	0.0192	0.0169	0.0156
		(0.0007)	(0.0005)	(0.0004)	(0.0004)	(0.0003)	(0.0003)
	JSA	0.0387	0.0269	0.0219	0.0191	0.0168	0.0155
		(0.0007)	(0.0005)	(0.0004)	(0.0004)	(0.0003)	(0.0003)
	MIIV	0.0389	0.0271	0.0220	0.0192	0.0168	0.0156
		(0.0007)	(0.0005)	(0.0004)	(0.0004)	(0.0003)	(0.0003)
	ML	0.0387	0.0270	0.0220	0.0191	0.0168	0.0156
		(0.0007)	(0.0005)	(0.0004)	(0.0004)	(0.0003)	(0.0003)
θ_9	JS	0.0481	0.0333	0.0267	0.0236	0.0206	0.0191
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0003)
	JSA	0.0483	0.0334	0.0267	0.0237	0.0206	0.0191
		(0.0009)	(0.0006)	(0.0005)	(0.0004)	(0.0004)	(0.0003)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	MIIV	0.0484 (0.0009)	0.0334 (0.0006)	0.0267 (0.0005)	0.0237 (0.0004)	0.0206 (0.0004)	0.0191 (0.0003)
	ML	0.0486 (0.0009)	0.0336 (0.0006)	0.0268 (0.0005)	0.0237 (0.0005)	0.0207 (0.0004)	0.0192 (0.0003)
θ_{10}	JS	0.0382 (0.0007)	0.0276 (0.0005)	0.0218 (0.0004)	0.0193 (0.0003)	0.0173 (0.0003)	0.0156 (0.0003)
	JSA	0.0380 (0.0007)	0.0275 (0.0005)	0.0218 (0.0004)	0.0192 (0.0003)	0.0173 (0.0003)	0.0156 (0.0003)
	MIIV	0.0384 (0.0007)	0.0276 (0.0005)	0.0219 (0.0004)	0.0192 (0.0003)	0.0173 (0.0003)	0.0156 (0.0003)
	ML	0.0382 (0.0007)	0.0276 (0.0005)	0.0218 (0.0004)	0.0192 (0.0003)	0.0173 (0.0003)	0.0156 (0.0003)
$\mathrm{Var}(\eta_1)$	JS	0.1717 (0.0032)	0.1191 (0.0021)	0.0991 (0.0019)	0.0857 (0.0015)	0.0793 (0.0014)	0.0741 (0.0014)
	JSA	0.1710 (0.0032)	0.1187 (0.0021)	0.0991 (0.0019)	0.0855 (0.0015)	0.0791 (0.0014)	0.0739 (0.0014)
	MIIV	0.1725 (0.0032)	0.1188 (0.0021)	0.0996 (0.0019)	0.0854 (0.0015)	0.0792 (0.0014)	0.0739 (0.0014)
	ML	0.1712 (0.0032)	0.1183 (0.0021)	0.0991 (0.0019)	0.0854 (0.0015)	0.0790 (0.0014)	0.0738 (0.0014)
$\mathrm{Var}(\zeta_2)$	JS	0.1223 (0.0023)	0.0841 (0.0016)	0.0699 (0.0012)	0.0592 (0.0011)	0.0540 (0.0010)	0.0487 (0.0009)
	JSA	0.1213 (0.0023)	0.0833 (0.0016)	0.0695 (0.0012)	0.0591 (0.0011)	0.0539 (0.0010)	0.0485 (0.0009)
	MIIV	0.1236 (0.0023)	0.0839 (0.0016)	0.0697 (0.0013)	0.0592 (0.0011)	0.0540 (0.0011)	0.0484 (0.0009)
	ML	0.1212 (0.0023)	0.0828 (0.0016)	0.0693 (0.0012)	0.0590 (0.0011)	0.0537 (0.0010)	0.0483 (0.0009)
$\mathrm{Var}(\zeta_3)$	JS	0.0903 (0.0017)	0.0645 (0.0012)	0.0537 (0.0010)	0.0458 (0.0008)	0.0406 (0.0008)	0.0372 (0.0007)
	JSA	0.0895 (0.0017)	0.0642 (0.0012)	0.0536 (0.0010)	0.0458 (0.0008)	0.0406 (0.0008)	0.0371 (0.0007)
	MIIV	0.0919 (0.0017)	0.0648 (0.0012)	0.0541 (0.0010)	0.0461 (0.0008)	0.0406 (0.0007)	0.0371 (0.0007)
	ML	0.0891 (0.0017)	0.0642 (0.0012)	0.0535 (0.0010)	0.0457 (0.0008)	0.0405 (0.0007)	0.0369 (0.0007)

Note. Monte Carlo standard errors in parentheses. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood.

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
B_{21}	JS	1.03 (0.04)	0.96 (0.04)	0.94 (0.03)	0.93 (0.03)	0.96 (0.04)	0.95 (0.03)
	JSA	1.06 (0.04)	0.99 (0.04)	0.94 (0.03)	0.98 (0.04)	0.95 (0.04)	0.99 (0.03)
	MIIV	1.06 (0.04)	1.02 (0.04)	0.94 (0.04)	1.01 (0.04)	0.95 (0.04)	1.00 (0.04)
	ML	0.99 (0.03)	0.98 (0.04)	0.92 (0.04)	0.97 (0.04)	0.96 (0.04)	1.00 (0.04)
B_{31}	JS	1.24 (0.05)	1.11 (0.04)	1.06 (0.04)	1.04 (0.04)	1.01 (0.04)	1.00 (0.04)
	JSA	1.12 (0.04)	0.98 (0.04)	0.97 (0.03)	1.06 (0.04)	0.99 (0.04)	1.00 (0.04)
	MIIV	1.12 (0.04)	0.98 (0.04)	0.99 (0.03)	1.07 (0.04)	1.01 (0.04)	1.00 (0.04)
	ML	1.00 (0.03)	0.94 (0.03)	0.96 (0.04)	1.00 (0.04)	1.01 (0.04)	0.97 (0.04)
B_{32}	JS	1.18 (0.05)	1.09 (0.04)	1.08 (0.04)	1.11 (0.04)	1.00 (0.04)	1.03 (0.04)
	JSA	1.06 (0.04)	1.02 (0.04)	1.03 (0.04)	1.08 (0.04)	0.97 (0.03)	1.09 (0.04)
	MIIV	1.10 (0.04)	1.02 (0.04)	1.04 (0.04)	1.05 (0.04)	0.99 (0.03)	1.09 (0.04)
	ML	0.97 (0.04)	0.95 (0.03)	1.06 (0.04)	1.03 (0.04)	1.02 (0.04)	1.07 (0.04)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
λ_2	JS	1.03 (0.04)	0.98 (0.04)	0.98 (0.03)	0.96 (0.03)	0.96 (0.03)	1.03 (0.04)
	JSA	1.08 (0.04)	0.99 (0.04)	0.95 (0.04)	0.96 (0.03)	0.97 (0.04)	1.01 (0.04)
	MIIV	1.06 (0.04)	1.01 (0.04)	0.97 (0.04)	0.99 (0.03)	0.96 (0.04)	1.01 (0.04)
	ML	1.04 (0.04)	1.00 (0.04)	0.95 (0.03)	0.96 (0.03)	0.96 (0.04)	0.99 (0.04)
λ_6	JS	1.04 (0.04)	0.99 (0.04)	0.99 (0.04)	1.00 (0.04)	1.04 (0.04)	1.05 (0.04)
	JSA	1.05 (0.04)	0.97 (0.03)	1.00 (0.04)	0.98 (0.03)	1.04 (0.04)	1.06 (0.04)
	MIIV	1.05 (0.04)	0.99 (0.04)	1.00 (0.04)	0.97 (0.03)	1.07 (0.04)	1.05 (0.04)
	ML	1.02 (0.04)	0.97 (0.03)	0.99 (0.04)	0.96 (0.03)	1.04 (0.04)	1.04 (0.04)
λ_{10}	JS	1.06 (0.04)	1.05 (0.04)	1.08 (0.04)	1.03 (0.04)	1.11 (0.04)	1.07 (0.04)
	JSA	1.03 (0.04)	1.06 (0.04)	1.03 (0.04)	0.98 (0.04)	1.08 (0.04)	1.06 (0.04)
	MIIV	1.07 (0.04)	1.09 (0.04)	1.04 (0.04)	1.04 (0.04)	1.08 (0.04)	1.09 (0.04)
	ML	1.01 (0.04)	1.06 (0.04)	1.02 (0.04)	0.99 (0.04)	1.08 (0.04)	1.07 (0.04)
θ_1	JS	1.03 (0.04)	0.98 (0.04)	0.99 (0.04)	0.97 (0.03)	0.96 (0.03)	1.07 (0.04)
	JSA	1.04 (0.04)	0.98 (0.04)	0.99 (0.04)	0.97 (0.03)	0.96 (0.03)	1.07 (0.04)
	MIIV	1.08 (0.04)	1.01 (0.04)	0.99 (0.04)	0.98 (0.04)	0.97 (0.04)	1.08 (0.04)
	ML	0.99 (0.04)	0.96 (0.04)	0.97 (0.03)	0.96 (0.03)	0.95 (0.03)	1.06 (0.04)
θ_2	JS	1.01 (0.04)	1.00 (0.04)	1.02 (0.04)	0.96 (0.03)	0.99 (0.03)	1.03 (0.04)
	JSA	1.02 (0.04)	1.01 (0.04)	1.01 (0.04)	0.96 (0.03)	1.01 (0.04)	1.03 (0.04)
	MIIV	0.97 (0.03)	0.98 (0.04)	0.99 (0.04)	0.95 (0.03)	0.99 (0.03)	1.02 (0.04)
	ML	0.96 (0.03)	0.97 (0.03)	0.99 (0.04)	0.94 (0.03)	0.99 (0.03)	1.01 (0.04)
θ_5	JS	1.10 (0.04)	1.06 (0.04)	1.00 (0.04)	0.93 (0.03)	1.03 (0.04)	1.01 (0.04)
	JSA	1.09 (0.04)	1.06 (0.04)	1.00 (0.04)	0.93 (0.03)	1.03 (0.04)	1.01 (0.04)
	MIIV	1.15 (0.04)	1.10 (0.04)	1.02 (0.04)	0.93 (0.03)	1.04 (0.04)	1.01 (0.04)
	ML	1.04 (0.04)	1.04 (0.04)	0.99 (0.04)	0.91 (0.03)	1.02 (0.04)	1.00 (0.04)
θ_6	JS	1.04 (0.04)	0.98 (0.04)	0.98 (0.03)	1.02 (0.03)	0.99 (0.04)	0.95 (0.04)
	JSA	1.02 (0.04)	1.00 (0.04)	0.97 (0.03)	1.00 (0.03)	0.98 (0.04)	0.94 (0.04)
	MIIV	0.98 (0.04)	0.96 (0.04)	0.96 (0.03)	1.00 (0.03)	0.98 (0.04)	0.94 (0.04)
	ML	0.97 (0.04)	0.95 (0.04)	0.96 (0.03)	0.99 (0.03)	0.97 (0.04)	0.94 (0.04)
θ_9	JS	1.04 (0.04)	1.01 (0.04)	0.99 (0.04)	1.01 (0.04)	1.01 (0.04)	1.00 (0.04)
	JSA	1.05 (0.04)	1.02 (0.04)	1.00 (0.04)	1.02 (0.04)	1.01 (0.04)	1.00 (0.04)
	MIIV	1.09 (0.04)	1.04 (0.04)	1.00 (0.04)	1.03 (0.04)	1.02 (0.04)	1.01 (0.04)
	ML	1.01 (0.04)	0.99 (0.04)	0.98 (0.04)	1.00 (0.04)	1.00 (0.04)	0.99 (0.04)
θ_{10}	JS	1.06 (0.04)	0.95 (0.04)	1.01 (0.04)	1.06 (0.04)	0.99 (0.04)	0.92 (0.03)
	JSA	1.06 (0.04)	0.95 (0.04)	1.00 (0.04)	1.03 (0.04)	1.00 (0.04)	0.93 (0.03)
	MIIV	1.02 (0.04)	0.94 (0.04)	1.00 (0.04)	1.03 (0.04)	0.99 (0.04)	0.92 (0.03)
	ML	1.00 (0.04)	0.93 (0.04)	0.99 (0.04)	1.02 (0.04)	0.98 (0.04)	0.92 (0.03)
$\mathrm{Var}(\eta_1)$	JS	1.02 (0.04)	0.95 (0.04)	1.00 (0.04)	0.96 (0.04)	0.96 (0.04)	1.00 (0.03)
	JSA	1.05 (0.04)	0.96 (0.04)	1.01 (0.04)	0.98 (0.04)	0.96 (0.04)	1.01 (0.03)
	MIIV	1.19 (0.04)	1.03 (0.04)	1.07 (0.04)	1.04 (0.04)	0.97 (0.04)	1.04 (0.04)
	ML	1.02 (0.04)	0.97 (0.04)	1.00 (0.04)	0.98 (0.03)	0.94 (0.03)	1.00 (0.03)
$\mathrm{Var}(\zeta_2)$	JS	1.08 (0.04)	0.99 (0.04)	1.02 (0.04)	0.98 (0.04)	1.01 (0.04)	0.99 (0.04)
	JSA	1.07 (0.04)	0.99 (0.04)	1.01 (0.04)	0.99 (0.04)	1.03 (0.04)	0.99 (0.04)
	MIIV	1.17 (0.04)	1.04 (0.04)	1.00 (0.04)	1.01 (0.04)	1.06 (0.04)	1.01 (0.04)
	ML	1.02 (0.04)	0.97 (0.04)	0.98 (0.04)	0.98 (0.04)	1.03 (0.04)	0.99 (0.04)
$\mathrm{Var}(\zeta_3)$	JS	1.08 (0.04)	1.05 (0.04)	1.04 (0.04)	1.00 (0.04)	1.04 (0.04)	1.02 (0.04)
	JSA	1.03 (0.04)	1.01 (0.04)	1.03 (0.04)	0.99 (0.03)	1.02 (0.04)	1.02 (0.04)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	MIIV	1.11 (0.04)	1.06 (0.04)	1.05 (0.04)	1.01 (0.04)	1.02 (0.04)	1.01 (0.04)
	ML	0.95 (0.03)	0.98 (0.04)	1.00 (0.04)	0.97 (0.03)	1.01 (0.04)	0.99 (0.04)

Note. Monte Carlo standard errors in parentheses. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood.

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
B_{21}	JS	0.94 (0.04)	0.98 (0.04)	1.02 (0.04)	0.98 (0.04)	1.04 (0.04)	0.94 (0.03)
	JSA	0.97 (0.04)	1.00 (0.04)	1.02 (0.04)	0.99 (0.04)	1.01 (0.04)	0.94 (0.03)
	MIIV	0.98 (0.04)	1.00 (0.04)	1.02 (0.04)	1.00 (0.04)	1.00 (0.04)	0.95 (0.03)
	ML	0.96 (0.03)	1.00 (0.04)	1.00 (0.04)	0.99 (0.04)	1.02 (0.04)	0.91 (0.03)
B_{31}	JS	0.96 (0.04)	0.96 (0.04)	1.00 (0.04)	1.06 (0.04)	0.96 (0.04)	1.03 (0.04)
	JSA	0.95 (0.04)	1.00 (0.04)	1.00 (0.04)	1.05 (0.04)	0.96 (0.04)	1.00 (0.04)
	MIIV	0.98 (0.04)	1.01 (0.04)	1.05 (0.04)	1.05 (0.04)	0.96 (0.04)	0.99 (0.04)
	ML	0.94 (0.04)	0.94 (0.03)	0.99 (0.04)	1.08 (0.04)	0.94 (0.03)	0.95 (0.04)
B_{32}	JS	0.95 (0.04)	0.95 (0.04)	1.06 (0.04)	0.99 (0.04)	0.98 (0.03)	1.02 (0.04)
	JSA	0.92 (0.03)	0.97 (0.04)	1.02 (0.04)	1.00 (0.04)	0.98 (0.03)	0.98 (0.04)
	MIIV	0.94 (0.03)	0.98 (0.04)	1.04 (0.04)	1.01 (0.04)	0.98 (0.03)	0.98 (0.03)
	ML	0.93 (0.03)	0.96 (0.04)	1.02 (0.04)	1.01 (0.04)	0.98 (0.04)	0.98 (0.04)
λ_2	JS	1.10 (0.04)	1.07 (0.04)	1.00 (0.04)	1.04 (0.04)	0.97 (0.04)	1.01 (0.04)
	JSA	1.10 (0.04)	1.06 (0.04)	1.00 (0.04)	1.02 (0.04)	0.97 (0.04)	1.01 (0.04)
	MIIV	1.12 (0.04)	1.09 (0.04)	1.02 (0.04)	1.03 (0.04)	0.98 (0.04)	1.02 (0.04)
	ML	1.09 (0.04)	1.07 (0.04)	0.99 (0.04)	1.02 (0.04)	0.97 (0.04)	1.01 (0.04)
λ_6	JS	0.93 (0.03)	0.97 (0.04)	1.00 (0.03)	1.00 (0.04)	0.99 (0.04)	0.98 (0.04)
	JSA	0.95 (0.04)	0.96 (0.04)	0.99 (0.04)	1.00 (0.04)	0.98 (0.04)	0.99 (0.04)
	MIIV	1.01 (0.04)	0.97 (0.04)	1.00 (0.04)	0.99 (0.04)	0.98 (0.04)	1.00 (0.04)
	ML	0.94 (0.03)	0.96 (0.04)	0.99 (0.04)	0.99 (0.04)	0.97 (0.04)	0.99 (0.04)
λ_{10}	JS	1.02 (0.04)	1.03 (0.04)	0.97 (0.03)	0.97 (0.04)	1.01 (0.04)	0.98 (0.04)
	JSA	0.99 (0.04)	1.02 (0.04)	0.96 (0.04)	0.97 (0.04)	0.99 (0.04)	0.99 (0.04)
	MIIV	1.01 (0.04)	1.03 (0.04)	0.98 (0.04)	0.97 (0.04)	1.00 (0.04)	1.00 (0.04)
	ML	0.99 (0.04)	1.02 (0.04)	0.96 (0.04)	0.97 (0.04)	1.00 (0.04)	1.00 (0.04)
θ_1	JS	0.98 (0.04)	1.00 (0.04)	1.01 (0.04)	0.96 (0.03)	1.00 (0.04)	0.92 (0.03)
	JSA	0.99 (0.04)	1.00 (0.04)	1.02 (0.04)	0.97 (0.03)	1.00 (0.04)	0.92 (0.03)
	MIIV	0.98 (0.04)	1.00 (0.04)	1.02 (0.04)	0.97 (0.03)	1.00 (0.04)	0.92 (0.03)
	ML	0.95 (0.03)	0.98 (0.03)	1.00 (0.04)	0.96 (0.03)	0.99 (0.04)	0.91 (0.03)
θ_2	JS	1.05 (0.04)	0.96 (0.03)	1.00 (0.04)	0.93 (0.03)	1.01 (0.04)	0.97 (0.04)
	JSA	1.04 (0.04)	0.97 (0.03)	1.00 (0.04)	0.93 (0.03)	1.00 (0.04)	0.97 (0.04)
	MIIV	1.00 (0.04)	0.94 (0.03)	0.99 (0.03)	0.92 (0.03)	1.00 (0.04)	0.96 (0.04)
	ML	1.00 (0.04)	0.94 (0.03)	0.98 (0.03)	0.92 (0.03)	1.00 (0.04)	0.97 (0.04)
θ_5	JS	0.99 (0.04)	1.01 (0.04)	1.00 (0.04)	0.98 (0.04)	0.93 (0.04)	1.02 (0.04)
	JSA	0.99 (0.04)	1.02 (0.04)	1.01 (0.04)	0.98 (0.04)	0.93 (0.04)	1.03 (0.04)
	MIIV	0.99 (0.04)	1.02 (0.04)	1.01 (0.04)	0.98 (0.04)	0.93 (0.04)	1.02 (0.04)
	ML	0.96 (0.04)	1.00 (0.04)	1.00 (0.03)	0.97 (0.04)	0.92 (0.04)	1.02 (0.04)
θ_6	JS	0.96 (0.03)	0.99 (0.03)	1.00 (0.04)	0.98 (0.04)	1.01 (0.04)	0.99 (0.04)
	JSA	0.95 (0.03)	0.99 (0.03)	0.99 (0.04)	0.98 (0.04)	1.02 (0.04)	0.99 (0.04)
	MIIV	0.92 (0.03)	0.97 (0.03)	0.98 (0.03)	0.97 (0.04)	1.01 (0.04)	0.99 (0.04)
	ML	0.91 (0.03)	0.97 (0.03)	0.98 (0.03)	0.97 (0.04)	1.01 (0.04)	0.98 (0.04)
θ_9	JS	0.94 (0.03)	0.99 (0.04)	1.02 (0.04)	0.97 (0.04)	1.03 (0.04)	0.99 (0.03)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	JSA	0.94 (0.03)	0.99 (0.04)	1.02 (0.04)	0.97 (0.04)	1.03 (0.04)	0.99 (0.03)
	MIIV	0.94 (0.03)	0.99 (0.04)	1.02 (0.04)	0.97 (0.04)	1.03 (0.04)	0.99 (0.03)
	ML	0.91 (0.03)	0.97 (0.04)	1.01 (0.04)	0.96 (0.04)	1.02 (0.04)	0.99 (0.03)
θ_{10}	JS	1.00 (0.04)	0.96 (0.03)	1.01 (0.04)	0.98 (0.03)	0.97 (0.03)	0.99 (0.04)
	JSA	1.00 (0.04)	0.96 (0.03)	1.01 (0.04)	0.98 (0.03)	0.96 (0.03)	0.99 (0.03)
	MIIV	0.96 (0.03)	0.94 (0.03)	1.00 (0.04)	0.97 (0.03)	0.96 (0.03)	0.98 (0.03)
	ML	0.95 (0.03)	0.94 (0.03)	1.00 (0.04)	0.97 (0.03)	0.95 (0.03)	0.98 (0.03)
$\mathrm{Var}(\eta_1)$	JS	1.06 (0.04)	1.08 (0.04)	1.06 (0.04)	1.07 (0.04)	0.99 (0.04)	0.94 (0.04)
	JSA	1.07 (0.04)	1.09 (0.04)	1.06 (0.04)	1.07 (0.04)	0.99 (0.04)	0.94 (0.04)
	MIIV	1.13 (0.04)	1.13 (0.04)	1.07 (0.04)	1.09 (0.04)	1.00 (0.04)	0.95 (0.04)
	ML	1.06 (0.04)	1.09 (0.04)	1.05 (0.04)	1.07 (0.04)	0.99 (0.04)	0.95 (0.04)
$\mathrm{Var}(\zeta_2)$	JS	0.94 (0.03)	1.00 (0.04)	0.97 (0.03)	1.00 (0.04)	0.97 (0.04)	0.99 (0.04)
	JSA	0.94 (0.03)	1.01 (0.04)	0.97 (0.03)	1.00 (0.04)	0.97 (0.04)	0.99 (0.04)
	MIIV	0.97 (0.03)	1.02 (0.04)	0.98 (0.03)	1.01 (0.04)	0.97 (0.04)	1.01 (0.04)
	ML	0.92 (0.03)	1.00 (0.04)	0.96 (0.03)	0.99 (0.04)	0.96 (0.04)	1.00 (0.04)
$\mathrm{Var}(\zeta_3)$	JS	1.07 (0.04)	1.02 (0.04)	0.98 (0.03)	1.01 (0.04)	1.02 (0.04)	1.02 (0.04)
	JSA	1.04 (0.04)	1.01 (0.04)	0.97 (0.03)	1.00 (0.04)	1.02 (0.04)	1.02 (0.04)
	MIIV	1.05 (0.04)	1.02 (0.04)	0.97 (0.03)	0.99 (0.04)	1.02 (0.04)	1.02 (0.04)
	ML	1.01 (0.04)	0.99 (0.04)	0.96 (0.03)	0.99 (0.03)	1.01 (0.04)	1.01 (0.04)

Note. Monte Carlo standard errors in parentheses. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood.

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
B_{21}	JS	.948 (.006)	.938 (.006)	.939 (.006)	.942 (.006)	.948 (.006)	.939 (.006)
	JSA	.954 (.005)	.947 (.006)	.936 (.006)	.942 (.006)	.942 (.006)	.946 (.006)
	MIIV	.952 (.006)	.956 (.005)	.937 (.006)	.948 (.006)	.945 (.006)	.953 (.005)
	ML	.946 (.006)	.946 (.006)	.933 (.006)	.942 (.006)	.947 (.006)	.943 (.006)
B_{31}	JS	.984 (.003)	.967 (.005)	.961 (.005)	.961 (.005)	.959 (.005)	.956 (.005)
	JSA	.958 (.005)	.944 (.006)	.944 (.006)	.954 (.005)	.953 (.005)	.951 (.006)
	MIIV	.963 (.005)	.955 (.005)	.956 (.005)	.959 (.005)	.952 (.006)	.946 (.006)
	ML	.956 (.005)	.943 (.006)	.944 (.006)	.948 (.006)	.949 (.006)	.949 (.006)
B_{32}	JS	.966 (.005)	.957 (.005)	.963 (.005)	.959 (.005)	.959 (.005)	.955 (.005)
	JSA	.952 (.006)	.955 (.005)	.951 (.006)	.954 (.005)	.949 (.006)	.955 (.005)
	MIIV	.958 (.005)	.950 (.006)	.953 (.005)	.955 (.005)	.948 (.006)	.962 (.005)
	ML	.943 (.006)	.944 (.006)	.951 (.006)	.955 (.005)	.957 (.005)	.957 (.005)
λ_2	JS	.934 (.006)	.937 (.006)	.939 (.006)	.939 (.006)	.947 (.006)	.951 (.006)
	JSA	.938 (.006)	.939 (.006)	.933 (.006)	.941 (.006)	.943 (.006)	.947 (.006)
	MIIV	.884 (.008)	.913 (.007)	.919 (.007)	.922 (.007)	.930 (.007)	.931 (.007)
	ML	.948 (.006)	.944 (.006)	.944 (.006)	.944 (.006)	.943 (.006)	.948 (.006)
λ_6	JS	.934 (.006)	.934 (.006)	.948 (.006)	.955 (.005)	.961 (.005)	.952 (.006)
	JSA	.930 (.007)	.932 (.007)	.953 (.005)	.949 (.006)	.955 (.005)	.959 (.005)
	MIIV	.892 (.008)	.900 (.008)	.925 (.007)	.933 (.006)	.943 (.006)	.943 (.006)
	ML	.942 (.006)	.942 (.006)	.953 (.005)	.947 (.006)	.954 (.005)	.962 (.005)
λ_{10}	JS	.946 (.006)	.946 (.006)	.957 (.005)	.944 (.006)	.958 (.005)	.952 (.006)
	JSA	.943 (.006)	.947 (.006)	.952 (.006)	.941 (.006)	.957 (.005)	.952 (.006)
	MIIV	.892 (.008)	.922 (.007)	.932 (.007)	.942 (.006)	.946 (.006)	.950 (.006)
	ML	.950 (.006)	.954 (.005)	.953 (.005)	.949 (.006)	.961 (.005)	.954 (.005)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
θ_1	JS	.930 (.007)	.929 (.007)	.937 (.006)	.946 (.006)	.938 (.006)	.940 (.006)
	JSA	.938 (.006)	.934 (.006)	.941 (.006)	.948 (.006)	.946 (.006)	.941 (.006)
	MIIV	.933 (.006)	.928 (.007)	.938 (.006)	.946 (.006)	.942 (.006)	.941 (.006)
	ML	.932 (.007)	.937 (.006)	.937 (.006)	.947 (.006)	.944 (.006)	.942 (.006)
θ_2	JS	.937 (.006)	.943 (.006)	.945 (.006)	.939 (.006)	.951 (.006)	.956 (.005)
	JSA	.927 (.007)	.944 (.006)	.941 (.006)	.938 (.006)	.953 (.005)	.957 (.005)
	MIIV	.937 (.006)	.945 (.006)	.940 (.006)	.939 (.006)	.954 (.005)	.953 (.005)
	ML	.924 (.007)	.944 (.006)	.939 (.006)	.934 (.006)	.949 (.006)	.955 (.005)
θ_5	JS	.928 (.007)	.936 (.006)	.937 (.006)	.929 (.007)	.956 (.005)	.946 (.006)
	JSA	.931 (.007)	.942 (.006)	.942 (.006)	.928 (.007)	.953 (.005)	.947 (.006)
	MIIV	.925 (.007)	.938 (.006)	.937 (.006)	.927 (.007)	.954 (.005)	.945 (.006)
	ML	.930 (.007)	.941 (.006)	.939 (.006)	.927 (.007)	.953 (.005)	.946 (.006)
θ_6	JS	.942 (.006)	.940 (.006)	.945 (.006)	.954 (.005)	.948 (.006)	.941 (.006)
	JSA	.936 (.006)	.936 (.006)	.937 (.006)	.949 (.006)	.943 (.006)	.931 (.007)
	MIIV	.937 (.006)	.934 (.006)	.939 (.006)	.952 (.006)	.945 (.006)	.937 (.006)
	ML	.928 (.007)	.930 (.007)	.934 (.006)	.944 (.006)	.941 (.006)	.935 (.006)
θ_9	JS	.927 (.007)	.936 (.006)	.939 (.006)	.941 (.006)	.945 (.006)	.945 (.006)
	JSA	.927 (.007)	.938 (.006)	.942 (.006)	.942 (.006)	.945 (.006)	.948 (.006)
	MIIV	.927 (.007)	.932 (.006)	.939 (.006)	.943 (.006)	.947 (.006)	.944 (.006)
	ML	.935 (.006)	.940 (.006)	.943 (.006)	.944 (.006)	.945 (.006)	.946 (.006)
θ_{10}	JS	.940 (.006)	.936 (.006)	.947 (.006)	.953 (.005)	.947 (.006)	.936 (.006)
	JSA	.935 (.006)	.930 (.007)	.945 (.006)	.946 (.006)	.941 (.006)	.941 (.006)
	MIIV	.942 (.006)	.932 (.007)	.947 (.006)	.951 (.006)	.943 (.006)	.941 (.006)
	ML	.931 (.007)	.924 (.007)	.945 (.006)	.944 (.006)	.943 (.006)	.937 (.006)
$\mathrm{Var}(\eta_1)$	JS	.942 (.006)	.940 (.006)	.943 (.006)	.941 (.006)	.939 (.006)	.958 (.005)
	JSA	.952 (.006)	.942 (.006)	.944 (.006)	.947 (.006)	.941 (.006)	.959 (.005)
	MIIV	.964 (.005)	.955 (.005)	.957 (.005)	.957 (.005)	.938 (.006)	.955 (.005)
	ML	.945 (.006)	.936 (.006)	.938 (.006)	.940 (.006)	.941 (.006)	.957 (.005)
$\mathrm{Var}(\zeta_2)$	JS	.947 (.006)	.946 (.006)	.953 (.005)	.951 (.006)	.945 (.006)	.940 (.006)
	JSA	.948 (.006)	.945 (.006)	.949 (.006)	.947 (.006)	.943 (.006)	.941 (.006)
	MIIV	.968 (.005)	.958 (.005)	.959 (.005)	.945 (.006)	.959 (.005)	.951 (.006)
	ML	.934 (.006)	.942 (.006)	.943 (.006)	.939 (.006)	.944 (.006)	.945 (.006)
$\mathrm{Var}(\zeta_3)$	JS	.946 (.006)	.948 (.006)	.943 (.006)	.949 (.006)	.950 (.006)	.953 (.005)
	JSA	.940 (.006)	.940 (.006)	.943 (.006)	.947 (.006)	.945 (.006)	.952 (.006)
	MIIV	.961 (.005)	.951 (.006)	.951 (.006)	.948 (.006)	.955 (.005)	.949 (.006)
	ML	.909 (.007)	.926 (.007)	.929 (.007)	.941 (.006)	.944 (.006)	.943 (.006)

Note. Monte Carlo standard errors in parentheses. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood.

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
B_{21}	JS	.936 (.006)	.949 (.006)	.958 (.005)	.950 (.006)	.949 (.006)	.943 (.006)
	JSA	.944 (.006)	.952 (.006)	.957 (.005)	.951 (.006)	.945 (.006)	.939 (.006)
	MIIV	.949 (.006)	.950 (.006)	.955 (.005)	.954 (.005)	.949 (.006)	.948 (.006)
	ML	.949 (.006)	.953 (.005)	.945 (.006)	.951 (.006)	.949 (.006)	.935 (.006)
B_{31}	JS	.947 (.006)	.949 (.006)	.951 (.006)	.957 (.005)	.949 (.006)	.951 (.006)
	JSA	.942 (.006)	.951 (.006)	.946 (.006)	.961 (.005)	.949 (.006)	.947 (.006)
	MIIV	.943 (.006)	.950 (.006)	.955 (.005)	.961 (.005)	.948 (.006)	.951 (.006)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	ML	.938 (.006)	.944 (.006)	.947 (.006)	.959 (.005)	.943 (.006)	.943 (.006)
B_{32}	JS	.942 (.006)	.944 (.006)	.960 (.005)	.951 (.006)	.957 (.005)	.950 (.006)
	JSA	.932 (.006)	.950 (.006)	.956 (.005)	.946 (.006)	.949 (.006)	.947 (.006)
	MIIV	.936 (.006)	.948 (.006)	.955 (.005)	.948 (.006)	.952 (.006)	.953 (.005)
	ML	.941 (.006)	.948 (.006)	.949 (.006)	.945 (.006)	.943 (.006)	.950 (.006)
λ_2	JS	.949 (.006)	.963 (.005)	.941 (.006)	.949 (.006)	.950 (.006)	.957 (.005)
	JSA	.953 (.005)	.961 (.005)	.943 (.006)	.949 (.006)	.951 (.006)	.949 (.006)
	MIIV	.936 (.006)	.953 (.005)	.937 (.006)	.944 (.006)	.949 (.006)	.951 (.006)
	ML	.954 (.005)	.961 (.005)	.945 (.006)	.949 (.006)	.950 (.006)	.954 (.005)
λ_6	JS	.943 (.006)	.945 (.006)	.949 (.006)	.951 (.006)	.948 (.006)	.942 (.006)
	JSA	.945 (.006)	.943 (.006)	.947 (.006)	.953 (.005)	.943 (.006)	.945 (.006)
	MIIV	.932 (.007)	.937 (.006)	.944 (.006)	.944 (.006)	.937 (.006)	.942 (.006)
	ML	.949 (.006)	.946 (.006)	.947 (.006)	.947 (.006)	.943 (.006)	.944 (.006)
λ_{10}	JS	.947 (.006)	.947 (.006)	.947 (.006)	.947 (.006)	.951 (.006)	.943 (.006)
	JSA	.945 (.006)	.950 (.006)	.946 (.006)	.947 (.006)	.953 (.005)	.947 (.006)
	MIIV	.932 (.006)	.936 (.006)	.939 (.006)	.945 (.006)	.951 (.006)	.945 (.006)
	ML	.949 (.006)	.951 (.006)	.947 (.006)	.948 (.006)	.953 (.005)	.948 (.006)
θ_1	JS	.916 (.007)	.945 (.006)	.942 (.006)	.933 (.006)	.945 (.006)	.938 (.006)
	JSA	.919 (.007)	.946 (.006)	.946 (.006)	.938 (.006)	.947 (.006)	.939 (.006)
	MIIV	.918 (.007)	.945 (.006)	.947 (.006)	.937 (.006)	.947 (.006)	.937 (.006)
	ML	.918 (.007)	.943 (.006)	.943 (.006)	.937 (.006)	.947 (.006)	.939 (.006)
θ_2	JS	.936 (.006)	.941 (.006)	.950 (.006)	.933 (.006)	.943 (.006)	.947 (.006)
	JSA	.936 (.006)	.941 (.006)	.945 (.006)	.931 (.007)	.941 (.006)	.950 (.006)
	MIIV	.937 (.006)	.941 (.006)	.945 (.006)	.935 (.006)	.943 (.006)	.946 (.006)
	ML	.932 (.007)	.939 (.006)	.941 (.006)	.932 (.007)	.941 (.006)	.949 (.006)
θ_5	JS	.919 (.007)	.941 (.006)	.949 (.006)	.945 (.006)	.935 (.006)	.954 (.005)
	JSA	.922 (.007)	.944 (.006)	.951 (.006)	.948 (.006)	.935 (.006)	.953 (.005)
	MIIV	.921 (.007)	.945 (.006)	.948 (.006)	.946 (.006)	.938 (.006)	.954 (.005)
	ML	.921 (.007)	.943 (.006)	.948 (.006)	.947 (.006)	.939 (.006)	.953 (.005)
θ_6	JS	.919 (.007)	.943 (.006)	.945 (.006)	.945 (.006)	.951 (.006)	.946 (.006)
	JSA	.918 (.007)	.943 (.006)	.944 (.006)	.941 (.006)	.947 (.006)	.945 (.006)
	MIIV	.920 (.007)	.945 (.006)	.943 (.006)	.943 (.006)	.951 (.006)	.946 (.006)
	ML	.910 (.007)	.940 (.006)	.943 (.006)	.939 (.006)	.946 (.006)	.946 (.006)
θ_9	JS	.931 (.007)	.937 (.006)	.941 (.006)	.933 (.006)	.951 (.006)	.943 (.006)
	JSA	.930 (.007)	.937 (.006)	.941 (.006)	.934 (.006)	.952 (.006)	.943 (.006)
	MIIV	.932 (.006)	.941 (.006)	.942 (.006)	.935 (.006)	.951 (.006)	.943 (.006)
	ML	.932 (.007)	.939 (.006)	.941 (.006)	.935 (.006)	.954 (.005)	.943 (.006)
θ_{10}	JS	.936 (.006)	.941 (.006)	.940 (.006)	.945 (.006)	.943 (.006)	.940 (.006)
	JSA	.930 (.007)	.940 (.006)	.941 (.006)	.946 (.006)	.943 (.006)	.945 (.006)
	MIIV	.937 (.006)	.946 (.006)	.941 (.006)	.949 (.006)	.946 (.006)	.945 (.006)
	ML	.922 (.007)	.939 (.006)	.939 (.006)	.945 (.006)	.945 (.006)	.945 (.006)
$\mathrm{Var}(\eta_1)$	JS	.944 (.006)	.950 (.006)	.942 (.006)	.953 (.005)	.943 (.006)	.935 (.006)
	JSA	.946 (.006)	.953 (.005)	.941 (.006)	.954 (.005)	.944 (.006)	.937 (.006)
	MIIV	.967 (.005)	.963 (.005)	.951 (.006)	.961 (.005)	.948 (.006)	.939 (.006)
	ML	.940 (.006)	.955 (.005)	.938 (.006)	.954 (.005)	.946 (.006)	.939 (.006)
$\mathrm{Var}(\zeta_2)$	JS	.917 (.007)	.935 (.006)	.933 (.006)	.959 (.005)	.938 (.006)	.947 (.006)

Estimand	Estimator	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 600$
	JSA	.922 (.007)	.937 (.006)	.935 (.006)	.958 (.005)	.940 (.006)	.949 (.006)
	MIIV	.942 (.006)	.943 (.006)	.941 (.006)	.959 (.005)	.941 (.006)	.957 (.005)
	ML	.915 (.007)	.934 (.006)	.933 (.006)	.955 (.005)	.937 (.006)	.949 (.006)
$\text{Var}(\zeta_3)$	JS	.945 (.006)	.944 (.006)	.943 (.006)	.949 (.006)	.947 (.006)	.947 (.006)
	JSA	.941 (.006)	.941 (.006)	.941 (.006)	.947 (.006)	.947 (.006)	.948 (.006)
	MIIV	.955 (.005)	.947 (.006)	.947 (.006)	.949 (.006)	.949 (.006)	.949 (.006)
	ML	.924 (.007)	.933 (.006)	.934 (.006)	.945 (.006)	.946 (.006)	.948 (.006)

Note. Monte Carlo standard errors in parentheses. JS = scaling James–Stein, JSA = aggregated James–Stein, MIIV = model-implied instrumental variable, ML = maximum likelihood.

COLOPHON Typeset with Typst 0.15.0 from the authors' Quarto source. Template jrepstat.typ at commit 9469f38. Set in STIX Two Text, STIX Two Math and JetBrains Mono. Built 24 November 2026.