Power Studies For Two-Sample and Goodness-of-Fit Methods For Multivariate Data

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Abstract

We present the results of a large number of simulation studies regarding the power of various goodness-of-fit as well as nonparametric two-sample tests for multivariate data. In two dimensions this includes both continuous and discrete data, in higher dimensions continuous data only. In general no single method can be relied upon to provide good power: any one method may be quite good for some combination of null hypothesis and alternative and may fail badly for another. Based on the results of these studies we propose a fairly small number of methods chosen such that for any of the case studies included here at least one of the methods has good power, and we check these selections by leave-one-out cross-validation. In our case studies the aggregated MMD test as implemented in *MD2sample* is the best single method for the two-sample problem; it differs from the published MMDAgg test, and the result should not be assumed to carry over to it. For the goodness-of-fit problem in two dimensions, chi-square tests with a small number of bins are best, especially if the marginals are the same under the null hypothesis and the alternative. In five dimensions a hybrid test, a two-sample MMD test against a large Monte Carlo sample from the null hypothesis, has clearly higher power than the goodness-of-fit methods. The studies were carried out using the R packages *MD2sample* and *MDgof*, available from CRAN. All results, together with the true type I errors, are included in the packages, and the scripts that produce every table and figure are part of this paper.

1 Introduction

Both the goodness-of-fit (gof) and the nonparametric two-sample problem have histories going back a century, with many contributions by some of the most eminent statisticians. In the goodness-of-fit problem we have a sample $(\mathbf{x}_1, .., \mathbf{x}_n)$ drawn from a random vector $\mathbf{X}$ with probability distribution F, possibly with unknown parameters, and we wish to test $H_0 : \mathbf{X} \sim F$. In the two-sample problem we also have a second sample $(\mathbf{y}_1, .., \mathbf{y}_m)$ from some distribution G, and here we want to test $H_0 : F = G$, that is we want to test whether the two data sets were generated by the same (unspecified) distribution.

The literature on both of these problems is vast and steadily growing. Detailed discussions can be found in [D'Agostino and Stephens, 1986], [Thas, 2010], [Rayner et al., 2009]. For an introduction to Statistics and hypothesis testing in general see [Casella and Berger, 2002] or [Bickel and Doksum, 2015].

The power studies in this article were carried out using **R** programs in the packages *MD2sample* and *MDgof*, available from the CRAN website. In the case of univariate data many methods such as the Kolmogorov-Smirnov test are already implemented for both problems in base **R**. For multivariate data however the available software is much smaller, and most of the methods included in *MD2sample* and *MDgof* are not implemented anywhere else. The packages themselves are described in [Rolke, 2026a]; here we report the power studies carried out with them.

An earlier paper [Rolke, 2025] reported power studies for the multivariate two-sample methods. The present paper adds the goodness-of-fit and hybrid studies and the cross-validation of the recommended selections of methods.

The highlights of both those packages are:

- many methods are implemented for continuous data in any dimension and discrete data in two dimensions. Discrete data includes the case of histogram (aka discretized or binned) data.
- the methods are implemented using both *Rcpp* [Eddelbuettel et al., 2026] and parallel programming.
- the routines allow for a random sample size, assumed to come from a Poisson distribution.
- there are routines that allow the user to combine several tests and find a corrected p value.
- the routines can also use any other user-defined tests.
- the packages include routines to easily carry out power studies.
- they include routines to easily run a large number of power studies and thereby compare a new method's performance to those included in the packages.

Including tests for discrete data in two dimensions is useful in two ways: Of course discrete data is of interest in its own right, and there are no implementations in R at this time. It also makes it possible to apply the tests to very large continuous data sets via discretization. While a test for a continuous data set with (say) 10000 observations can be done in a matter of a few minutes, for larger data sets the calculations will be quite time consuming. Binning the data and then running the corresponding discrete tests however is quite fast. At least this is true for bivariate data. In more than two dimensions the number of bins grows rapidly, and the number of observations required would be extremely large. We therefore implemented discrete methods in two dimensions only.

There are also situations where the underlying distribution is continuous but the data is collected in binned form. This is for example often the case for data from high energy physics experiments and from astronomy because of finite detector resolution. In this situation the theoretical distribution is continuous but the data is discrete.

For most tests in the two-sample problem p-values are found via the permutation method, and in the goodness-of-fit case via simulation (aka parametric bootstrap). The exceptions are the chi-square tests for continuous data and some two-sample tests that use large sample theory, as described in the section on methods.

2 The Types of Problems Included in this Study

In general goodness-of-fit and two sample methods have found many uses. For example, in high energy physics they are sometimes used for discoveries. Our aim here is to use them for their original purpose. That is, a researcher has a data set and wishes to do some inference. The method he wishes to employ requires a probability model, and he has one he believes will fit. But before continuing his analysis he wishes to make certain that his model, in the immortal words of George Box, is "good enough".

The kinds of situations in which these methods will be useful are:

- **Goodness-of-Fit Problem - Continuous Data**: We have a sample x of size of n. F is a continuous probability distribution, which may depend on unknown parameters. We want to test $X \sim F$.
- **Goodness-of-Fit Problem - Discrete Data**: We have a set of values *valsx* of one variable and *valsy* of the other variable, and a vector of counts x. F is a discrete probability distribution, which may depend on unknown parameters. We want to test $X \sim F$.
- **Two-sample Problem - Continuous Data**: We have a sample x of size of n, drawn from some unknown continuous probability distribution F, and a sample y of size m, drawn from some unknown continuous probability distribution G. We want to test $F = G$.
- **Two-sample Problem - Discrete Data**: We have a set of values *valsx* of one variable and *valsy* of the other variable and vectors of counts x and y, drawn from some unknown discrete probability distributions F and G. We want to test $F = G$.

For a goodness-of-fit test we also have the case of a simple test, where the distribution function is fully specified (say $\mathbf{X} \sim \mathbf{N}(\mu, \mathbf{\Sigma})$) or the case of a composite test where some parameters are unspecified and have to be estimated from the data.

3 Goodness-of-fit/two-sample hybrid tests

There is another (quite popular) way to carry out a goodness-of-fit test. In these tests one generates a second data set y under the null hypothesis, using a parametric bootstrap approach if parameters are estimated. Then one carries out a two-sample test.

There are a number of reasons one might do this:

- One can now employ any two-sample method.
- The actual distribution function F might not be calculable, maybe because it requires high-dimensional integration.

This approach does raise an additional issue. If generating data from F is fast, how large a sample should be generated, relative to the sample size of the real data? Generally one would expect the power to increase with a larger data set.

In this paper we will include this hybrid method with two sample sizes for the MC data set, one equal to the size of the real data and another five times as large. We write nMC for the ratio of the size of the MC data set to the size of the real data set, so nMC=1 and nMC=5.

4 Highlights of the Results

Detailed results of our studies can be found later, here are some highlights:

- While there is a plethora of methods for the two-sample problem, multivariate goodness-of-fit tests are much rarer. In fact we found only three in the literature that fit our requirements (see section 5), namely the tests by Bickel and Breiman, by Bakshaev and Rudzkis and the Kernel Stein Discrepancy. Some more methods are also included in *MDgof* but work only in two dimensions.

- As one would expect, no single method is uniformly better than all the others.
- If the null hypothesis is true at least for the marginal distributions, that is, if those are indeed the same under the null hypothesis and the alternative, then in both the goodness-of-fit and the two-sample problem the classic chi-square test (with just a few bins!) has quite impressive power, often much better than any other method. Unfortunately chi-square tests require binning and are really only feasible in two dimensions.
- Hybrid methods (doing a goodness-of-fit test by generating a second MC data set) require a larger MC data set than the real data in order to be competitive. In two dimensions the goodness-of-fit methods (especially the chi-square tests) remain better even then, so as long as it is possible to do a classic goodness-of-fit test, these are preferred. In five dimensions, however, the MMD test with an MC data set five times the size of the real data has clearly higher power than any of the goodness-of-fit methods.
- In a hybrid test the power can usually be improved by using a larger MC data set, if this is feasible. Just how much larger is a difficult question to answer.
- For continuous data and a two-sample problem (or a hybrid problem if it is not possible to do a goodness-of-fit test because calculating values from the cumulative distribution function is not possible) the aggregated MMD test as implemented in *MD2sample* is the single best option in our case studies. This implementation differs from MMDAgg [Schrab et al., 2023] in how the statistics for the different bandwidths are combined and in how the p value is found (see section 5.4.6), so this result should not be assumed to carry over to MMDAgg or to other implementations of the MMD test. For a goodness-of-fit problem in 2 dimensions a chi-square test with a small number of bins works well, especially in cases where the distributions of the marginals are indeed those from the null hypothesis. In higher dimensions several tests have about equal power.

Based on the results of these simulation studies, and so for problems similar to those included here, our recommendations for the best methods are:

- Goodness-of-fit, continuous data, dim=2: a chi-square test with a small grid (say 5 by 5), Fasano-Franceschini, Ripley's K, the Kernel Stein Discrepancy and the simplified versions of Kuiper's test and the Anderson-Darling test implemented in *MDgof*, see the section on Methods. qAD should be used only in two dimensions; in five dimensions it is biased for several alternatives (see section 10).
- Goodness-of-fit, continuous data, dim $>$ 2: Bakshaev-Rudzkis and the simplified versions of the Kolmogorov-Smirnov test, Kuiper's test and the Cramer-vonMises test. If it is feasible to generate a large MC data set, the hybrid MMD test (see below) has higher power.
- Goodness-of-fit, discrete data: Kullback-Leibler, classic Pearson chi-square, Kuiper, Kolmogorov-Smirnov and Anderson-Darling.
- Goodness-of-fit + hybrid methods, dim=2: the goodness-of-fit methods listed above; among the hybrid methods MMD with an MC data set 5 times the size of the real data set.
- Goodness-of-fit + hybrid methods, dim $>$ 2: MMD and Aslan-Zech (the two methods with the highest mean power), both with an MC data set 5 times the size of the real data set, as well as qCvM.
- Two-sample, continuous data, dim=2: MMD, Aslan-Zech and a chi-square test.
- Two-sample, continuous data, dim $>$ 2: MMD, 5-nearest-neighbor and Aslan-Zech.
- Two-sample, discrete data, dim=2: a chi-square test, Kuiper and Baringhaus-Franz or Aslan-Zech.

Here and throughout the paper MMD refers to the aggregated MMD test as implemented in *MD2sample*, both for the two-sample and for the hybrid tests.

5 The Methods

5.1 Chi-square tests

5.1.1 Continuous data - goodness-of-fit

Because of the need for binning chi-square tests are implemented for bivariate data only. The data is binned into a rectangular grid, with n_1 bins in the x direction and n_2 bins into the y direction. The simulations all use $n_1 = n_2 = 5$. The bins are either equal-space *ES* or equal probability *EP*. For ES the cut points of each coordinate are equally spaced between the smallest and the largest observation, for EP they are the quantiles of the marginal distributions under the null hypothesis (with estimated parameters if these are estimated). In both cases the outermost bins extend to the boundary of the support.

In all cases neighboring bins with low counts are joined until all bins have an expected count of at least 5 in the goodness-of-fit problems and an actual count of 5 in the two-sample problem. In all cases the p-values are found using the usual chi-square approximation, except in the goodness-of-fit discrete data case, where simulation is used.

If parameters have to be estimated, this is done via the user-provided routine *phat*. As long as the method of estimation used is consistent and efficient and the expected counts are large enough the chi-square statistic will have a chi-square distribution, with the degrees of freedom reduced by the number of estimated parameters, as shown by [Fisher, 1922] and [Fisher, 1924]. This requires that the parameters are estimated from the binned data. If they are estimated from the original data, for example by maximum likelihood, the null distribution of the statistic lies between chi-square distributions with $k - 1 - p$ and $k - 1$ degrees of freedom [Chernoff and Lehmann, 1954], where k is the number of bins and p the number of parameters, and so the test with $k-1-p$ degrees of freedom is somewhat liberal. In *MDgof* the parameters are therefore estimated by minimum chi-square: the estimate supplied by the user (usually found from the original data) is used to find the bins and as the starting value, and then Pearson's statistic is minimized over the parameters with the bins held fixed.

The formula for the test statistic is

$$\sum_{i,j} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where O_{ij} is the observed count in bin (i, j) and E_{ij} is the expected count under the null hypothesis.

5.1.2 Discrete data - goodness-of-fit

Here the binning is already given. In this case the p-value is found via simulation rather than the usual large sample theory because there are several tests using the same ingredients (O_{ij} and E_{ij}) but without a large sample theory, so using simulation is needed anyway. This also means that joining bins so that $E_{ij} > 5$ (say) is not necessary.

5.1.3 Continuous data - two-sample problem

Again using a binning scheme as above, but with the cut points found from the combined data set (equally spaced for ES, sample quantiles for EP), we now have the observed counts O_{ij} and M_{ij} for the two data sets in the $(i, j)^{th}$ bin. Let $N_1 = \sum O_{ij}$ and $N_2 = \sum M_{ij}$, then the chi-square test statistic is given by

$$\sum_{i,j} \frac{\left(\sqrt{N_2/N_1}\, O_{ij} - \sqrt{N_1/N_2}\, M_{ij}\right)^2}{O_{ij} + M_{ij}}$$

which is the same as Pearson's chi-square statistic for the $2 \times k$ table of the counts, with the expected counts found by combining the two data sets.

Under the null hypothesis this test statistic will have approximately a chi-square distribution with the degrees of freedom equal to the number of bins - 1.

5.1.4 Discrete data - two-sample problem

Similar to the continuous case with the binning already given. Here cells without any observations are not used.

5.1.5 Binning

There are two basic binning schemes, equal-size and equal-probability bins. Which is better depends on the case study. In many of those included here under the null hypothesis the marginals have a uniform distribution, and so these types of bins are the same.

As far as the number of bins is concerned, previous studies have shown that fairly few bins usually lead to better power, see for example [Rolke and Gutierrez Gongora, 2021]. Here is the result of a study that shows that the same is true for multivariate data. We use 20 of the case studies for continuous data in D=2 without estimation. We run the chi-square test with $i \times i = i^2$ bins, $i = 3, 5, 7, 9$. Figure 1 shows the powers, which clearly show that in almost all cases fewer bins are better. Note that the degrees of freedom are sometimes a bit smaller as in some case studies there are empty bins by design. In both *MDgof* and *MD2sample* the default is a 5×5 grid, but the user can change this.

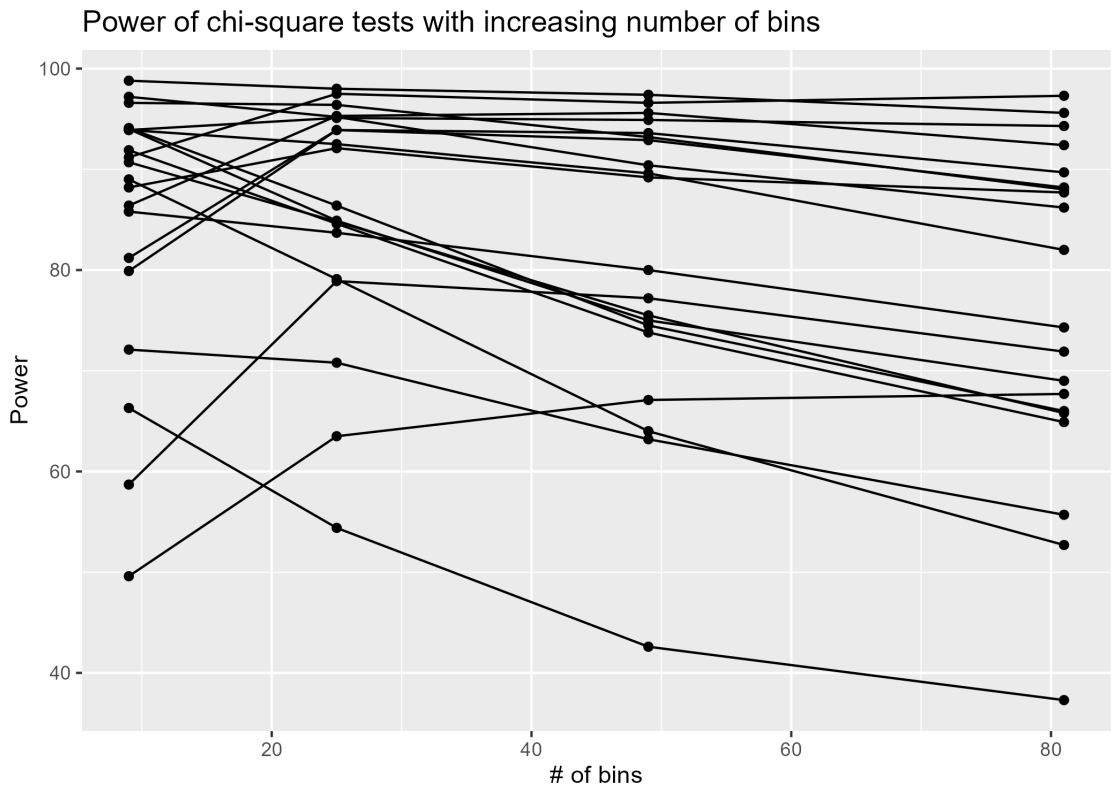

Figure 1: Power (in %) of the chi-square test against the number of bins $i \times i$, $i = 3, 5, 7, 9$. Each line is one of 20 case studies with continuous data in two dimensions without estimation.

5.2 Goodness-of-fit methods for continuous data

The goodness-of-fit problem is one of the oldest problems in Statistics, and there is a plethora of methods for univariate data. Surprisingly, this is not true for multivariate data, where there are only very few methods that have been discussed in the literature. To be precise, we are looking for methods with the following features:

1. The method should be a true omnibus test, that is it should not be designed for a specific distribution (like the multivariate normal) either under the null hypothesis or the alternative.
2. The method should allow for parameter estimation.
3. The method should not require the generation of an MC data set and run a subsequent two-sample test.
4. we do not require a large sample theory but can always find p-values via simulation.

To understand why there are so few such methods, let us see what it would mean to extend one of the classic univariate goodness-of-fit tests to multivariate data, namely the Kolmogorov-Smirnov test. It is based on the largest absolute deviation of the cumulative and the empirical distribution functions:

$$\psi(F, \hat{F}) = \max \left\{ |F(\mathbf{x}) - \hat{F}(\mathbf{x})| : \mathbf{x} \in \mathbf{R^d} \right\}$$

Now in principle this maximum is taken over all of $\mathbf{R^d}$, however in one dimension it is easy to see that the maximum will always be obtained at one of the data points, and therefore can be found with n evaluations of F. Unfortunately in higher dimensions this is no longer true. Here the maximum can occur at any point whose coordinates are any combination of the coordinates of the points in the data set, and there are n^d such points. So even for modest n and d this is impossible.

Some attempts have been made to find short-cuts, for example the Fasano-Franceschini test described later. In *MDgof* we have implemented a very simple version, where (as in one dimension) we just find the maximum among the data points. Here $F(\mathbf{x}) = P(\mathbf{X} \leq \mathbf{x})$ is the probability of the lower-left orthant at $\mathbf{x}$ (componentwise $\leq$) and $\hat{F}(\mathbf{x})$ the proportion of observations in it. Clearly one would expect this method to have lower power than a "full" test, but at least it can be applied to reasonably large data sets. Several other methods also based on a comparison of the theoretical and the empirical distribution function have also been implemented in the same manner. As far as we know, these tests have not been discussed in the literature for data with more than one dimension, although they are easy to implement and can have good power, as our simulation studies have shown. They are:

5.2.1 Kolmogorov-Smirnov test (qKS)

$$TS = \max_i \max \left\{ |F(\mathbf{x}_i) - \hat{F}(\mathbf{x}_i)|, |F(\mathbf{x}_i) - \hat{F}(\mathbf{x}_i) + 1/n| \right\}$$

where the second term accounts for the jump of $\hat{F}$ at the data points, as in the univariate test.

The KS test was first proposed in [Kolmogorov, 1933] and [Smirnov, 1939]. We use the notation *qKS* (quick Kolmogorov-Smirnov) to distinguish the test implemented in *MDgof* from the full test.

5.2.2 Kuiper's test (qK)

This is a variation of the KS test proposed in [Kuiper, 1960]:

$$\psi(F, \hat{F}) = \max \left\{ F(\mathbf{x}) - \hat{F}(\mathbf{x}) : \mathbf{x} \in \mathbf{R^d} \right\} + \max \left\{ \hat{F}(\mathbf{x}) - F(\mathbf{x}) : \mathbf{x} \in \mathbf{R^d} \right\}$$

$$TS = \max \left\{ F(\mathbf{x}_i) - \hat{F}(\mathbf{x}_i) \right\} + \max \left\{ \hat{F}(\mathbf{x}_i) - F(\mathbf{x}_i) \right\}$$

5.2.3 Cramer-vonMises test (qCvM)

Another classic test based on

$$\psi(F, \hat{F}) = \int \left(F(\mathbf{x}) - \hat{F}(\mathbf{x}) \right)^2 dF(\mathbf{x})$$

is implemented with

$$TS = \sum_{i=1}^{n} \left(F(\mathbf{x}_i) - \hat{F}(\mathbf{x}_i) \right)^2$$

This test was first discussed in [Anderson, 1962].

5.2.4 Anderson-Darling test (qAD)

The Anderson-Darling test is based on

$$\psi(F,\hat{F}) = \int \frac{\left(F(\mathbf{x}) - \hat{F}(\mathbf{x})\right)^2}{F(\mathbf{x})[1-F(\mathbf{x})]} dF(\mathbf{x})$$

We use the test statistic

$$TS = \sum_{i=1}^{n} \frac{\left(F(\mathbf{x}_i) - \hat{F}(\mathbf{x}_i)\right)^2}{F(\mathbf{x}_i)[1-F(\mathbf{x}_i)]}$$

It was first proposed in [Anderson and Darling, 1952].

In the following we discuss the few tests that have been proposed in the literature:

5.2.5 Bickel-Breiman test (BB)

This test uses the density, not the cumulative distribution function.

Let $R_j = \min\{||\mathbf{x}_i - \mathbf{x}_j|| : 1 \le i \le n, i \ne j\}$ be the distance of $\mathbf{x}_j$ to its nearest neighbor. Let f be the density function under the null hypothesis and define

$$U_j = \exp\left[-n \int_{||\mathbf{x}-\mathbf{x}_j||<R_j} f(\mathbf{x}) d\mathbf{x}\right]$$

Then it can be shown that under the null hypothesis $U_1, .., U_n$ have approximately a uniform distribution on $[0,1]$, and a goodness-of-fit test for univariate data can be applied. This test was first discussed in [Bickel and Breiman, 1983]. In *MDgof* the integral is approximated by $f(\mathbf{x}_j)$ times the volume of the ball with radius R_j, and the test statistic is $\frac{1}{n}\sum_{j=1}^{n}\left(U_{(j)} - j/n\right)^2$, where $U_{(1)} \le .. \le U_{(n)}$ are the ordered values.

5.2.6 Bakshaev-Rudzkis test (BR)

This test proceeds by estimating the density via a kernel density estimator and then comparing it to the density specified in the null hypothesis. Details are discussed in [Bakshaev and Rudzkis, 2015]. In *MDgof* a Gaussian kernel density estimate $\hat{f}_h$ is found at points drawn from the null distribution, it is standardized by its mean and standard deviation under the null hypothesis (found by simulation), and the test statistic is the largest absolute standardized value over these points and the bandwidths $h = 0.3, 0.5, 0.8$.

5.2.7 Kernel Stein Discrepancy (KSD)

This test also uses the density f under the null hypothesis, through its score function $\mathbf{s}(\mathbf{x}) = \nabla \log f(\mathbf{x})$, which does not depend on the normalizing constant of f. With a kernel k the Stein kernel is

$$u(\mathbf{x},\mathbf{y}) = \mathbf{s}(\mathbf{x})'\mathbf{s}(\mathbf{y})k(\mathbf{x},\mathbf{y}) + \mathbf{s}(\mathbf{x})'\nabla_{\mathbf{y}}k(\mathbf{x},\mathbf{y}) + \mathbf{s}(\mathbf{y})'\nabla_{\mathbf{x}}k(\mathbf{x},\mathbf{y}) + \text{trace}\left(\nabla_{\mathbf{x}}\nabla_{\mathbf{y}}k(\mathbf{x},\mathbf{y})\right)$$

and the test statistic is $\frac{1}{n^2}\sum_{i,j} u(\mathbf{x}_i, \mathbf{x}_j)$, an estimate of the squared Kernel Stein Discrepancy, which is zero if and only if the data comes from f [Liu et al., 2016], [Chwialkowski et al., 2016]. *MDgof* uses a Gaussian kernel with the bandwidth chosen by the median heuristic, and finds the score function by numerical differentiation of $\log f$.

5.2.8 Rosenblatt transforms

A number of tests have been proposed based on the Rosenblatt transform. This is a generalization of the probability integral transform. It transforms a random vector $(X_1, .., X_d)$ into $(U_1, .., U_d)$, where $U_i \sim U[0,1]$ and U_i is independent of U_j. It works as follows:

$$\begin{aligned}
U_1 &= F_{X_1}(x_1) \\
U_2 &= F_{X_2|X_1}(x_2|x_1) \\
&... \\
U_d &= F_{X_d|X_1,..,X_{d-1}}(x_d|x_1, .., x_{d-1})
\end{aligned}$$

and so requires knowledge of the conditional distribution functions. In our case of a goodness-of-fit test, however, these will generally not be known. One can show, though, that

$$\begin{aligned}
F_{X_1}(x_1) &= F(x_1, \infty, .., \infty) \\
F_{X_2|X_1}(x_2|x_1) &= \frac{\frac{d}{dx_1}F(x_1, x_2, \infty, .., \infty)}{\frac{d}{dx_1}F(x_1, \infty, ..\infty)} \\
&... \\
F_{X_d|X_1,..,X_{d-1}}(x_d|x_1, .., x_{d-1}) &= \frac{\frac{d^{d-1}}{dx_1 x_2 .. x_{d-1}}F(x_1, .., x_d)}{\frac{d^{d-1}}{dx_1 x_2 .. x_{d-1}}F(x_1, \infty, .., \infty)}
\end{aligned}$$

Unfortunately for general cdf F, these derivatives will have to be found numerically, and for $d > 2$ this would not be feasible because of issues with calculation times and numerical instabilities. For these reasons these methods are only implemented for bivariate data.

MDgof includes two tests based on the Rosenblatt transform:

5.2.9 Fasano-Franceschini test (FF)

This is a two-dimensional version of the Kolmogorov-Smirnov test discussed in [Fasano and Franceschini, 1987]. In *MDgof* it is applied after the Rosenblatt transform: for each data point the proportions of observations in the four quadrants defined by that point are compared to their probabilities under independent uniforms, and the test statistic is the largest absolute difference.

5.2.10 Ripley's K test (RK)

This test is based on Ripley's K function, the expected number of further observations within a radius r of a given observation, divided by the intensity. After the Rosenblatt transform (if the null hypothesis is true)

the data is supposed to be independent uniforms, and so $K(r) = \pi r^2$, or $L(r) = \sqrt{K(r)/\pi} = r$. The test statistic is the largest absolute difference between the estimated L function (with border correction) and r for $0.05 \leq r \leq 0.4$. This test was proposed in [Ripley, 1976]. The test is implemented in *MDgof* using the R library *spatstat* [Baddeley and Turner, 2005].

5.3 Goodness-of-fit methods for discrete data

In the case of discrete (as well as histogram) data methods based on comparing the theoretical and the empirical distribution function actually work again as in one dimension because now the points are known and it is the counts that are random. For this reason the implementations of the Kolmogorov-Smirnov test (KS), Kuiper's test (K), Cramer-vonMises test (CvM) and Anderson-Darling test (AD) are straightforward and are no longer short-cuts. For CvM and AD the sums are weighted by the probabilities of the cells under the null hypothesis.

In addition *MDgof* has a number of tests that similarly to the chi-square test compare the observed and the expected counts. They are

5.3.1 Pearson's chi-square (P)

$$\sum_{ij} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

5.3.2 Total Variation (TV)

$$\frac{1}{n} \sum_{ij} |O_{ij} - E_{ij}|$$

5.3.3 Kullback-Leibler (KL)

$$\frac{1}{n} \sum_{ij} O_{ij} \log \left(O_{ij} / E_{ij} \right)$$

5.3.4 Hellinger (H)

$$\frac{1}{n} \sum_{ij} \left(\sqrt{O_{ij}} - \sqrt{E_{ij}} \right)^2$$

5.4 Two-sample methods for continuous data

5.4.1 Methods based on empirical distribution functions

These are the classic methods Kolmogorov-Smirnov (KS) [Kolmogorov, 1933], [Smirnov, 1939], Kuiper (K) [Kuiper, 1960], Cramer-vonMises (CvM) [Anderson, 1962] and Anderson-Darling (AD) [Anderson and Darling, 1952]. As in the goodness-of-fit case the empirical distribution functions are those of the lower-left orthants, and

they are compared at the points of the combined data set. In two or more dimensions the largest difference need not occur at one of these points, so these are also simplified versions of the full tests. AD uses the empirical distribution function of the combined data set in the weights.

5.4.2 Methods based on nearest neighbors

The test statistics are the average number of nearest neighbors of the **x** data set that are also from **x** plus the average number of nearest neighbors of the **y** data set that are also from **y** [Schilling, 1986]. *NN1* uses one nearest neighbor and $NN5$ uses 5.

Next we have a number of methods based on distances between observations. We denote by $||.||$ Euclidean distance.

5.4.3 Aslan-Zech test (AZ)

This test discussed in [Aslan and Zech, 2005] uses the test statistic

$$\frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m}\log(||x_i - y_j||) -$$
$$\frac{1}{n^2}\sum_{i=1}^{n}\sum_{i<j}\log(||x_i - x_j||) -$$
$$\frac{1}{m^2}\sum_{i=1}^{m}\sum_{i<j}\log(||y_i - y_j||)$$

5.4.4 Baringhaus-Franz test (BF)

Similar to the Aslan-Zech test, it uses the test statistic

$$\frac{nm}{n+m}\left[\frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m}||x_i - y_j|| -\right.$$
$$\frac{1}{n^2}\sum_{i=1}^{n}\sum_{i<j}||x_i - x_j|| -$$
$$\left.\frac{1}{m^2}\sum_{i=1}^{m}\sum_{i<j}||y_i - y_j||\right]$$

and was first proposed in [Baringhaus and Franz, 2004].

5.4.5 Biswas-Ghosh test (BG)

Another variation of a test based on Euclidean distance was discussed in [Biswas and Ghosh, 2014].

$$B_{xy} = \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} ||x_i - y_j||$$

$$B_{xx} = \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{i<j} ||x_i - x_j||$$

$$B_{yy} = \frac{2}{m(m-1)} \sum_{i=1}^{m} \sum_{i<j} ||y_i - y_j||$$

$$(B_{xx} - B_{xy})^2 + (B_{yy} - B_{xy})^2$$

5.4.6 Maximum Mean Discrepancy test (MMD)

The Maximum Mean Discrepancy (MMD) two-sample test is a nonparametric kernel-based test. It works by embedding the two distributions into a reproducing kernel Hilbert space (RKHS) and measuring the distance between their mean embeddings. If this distance (the MMD) is zero, the two distributions are identical for characteristic kernels such as the Gaussian kernel; larger values indicate a discrepancy. The aggregated version implemented in *MD2sample* computes the unbiased estimate of the squared MMD with a Gaussian kernel for 10 bandwidths (the median distance between the observations times 2^l, $-4 \leq l \leq 4$), standardizes each by an estimate of its standard deviation under the null hypothesis and uses the largest of these as the test statistic, with the p value found by the permutation method. This is not the MMDAgg test of [Schrab et al., 2023], which aggregates the tests for the individual bandwidths with a correction of their levels rather than using the largest standardized statistic, and the powers reported here should not be assumed to hold for MMDAgg. For detailed discussions see [Gretton et al., 2012], [Chwialkowski et al., 2016] and [Schrab et al., 2023].

The following methods find p values using a large sample theory.

5.4.7 Friedman-Rafsky test (FR)

This test is a multi-dimensional extension of the classic Wald-Wolfowitz test based on minimum spanning trees. It was discussed in [Friedman and Rafsky, 1979]. In *MD2sample* it is the same test as CF1 below.

5.4.8 Simple Nearest Neighbor test (NN0)

Similar to the nearest neighbor tests described earlier, it uses only the number of nearest neighbors (one per observation) that come from the same data set. The p value is found from a normal approximation, using the exact mean and variance of this number under random permutation of the labels [Schilling, 1986].

5.4.9 Chen-Friedman tests (CF1 to CF4)

These tests, discussed in [Chen and Friedman, 2017], are implemented in the *gTests* [Chen and Zhang, 2017] package. CF1 is the original edge-count test, CF2 the generalized, CF3 the weighted and CF4 the max-type edge-count test. CF3 is the same as CF1 if both data sets have the same size, as is the case in our studies.

5.4.10 Ball Divergence test (Ball)

A test described in [Pan et al., 2018] and implemented in the R package *Ball* [Zhu et al., 2021]. The p value is found from the limiting distribution of the test statistic.

5.5 Two-sample Problem - discrete data

Implemented for discrete data are versions of the Kolmogorov-Smirnov (KS), Kuiper (K), Cramer-vonMises (CvM), Anderson-Darling (AD), nearest neighbor (NN, a test for discrete data different from NN0), Aslan-Zech (AZ), Baringhaus-Franz (BF) tests as well as a chi-square test. For KS, K, CvM, AD, AZ and BF the test statistics are the same as those for continuous data applied to the individual observations, with the values of the grid as their coordinates. The NN test compares, for each cell, the counts in the four neighboring cells with their expected values found from the combined data set. Except for the chi-square test, the p values are found by simulation: the count of each cell of the combined data set is split between the two data sets by binomial sampling with success probability $N_1/(N_1 + N_2)$.

6 Case studies

The conclusions described in this paper regarding the powers of the various methods are based on numerous case studies. We have the following different types of studies:

1. goodness-of-fit or two-sample problem.
2. continuous or discrete data.
3. 2 or 5 dimensional data.
4. studies with marginals that have the same distribution, or not.
5. studies with or without parameter estimation (for goodness-of-fit problem).

For the goodness-of-fit problem there are 30 case studies each for cases with and without parameter estimation in two dimensions plus 30 cases for five dimensional data. Of these, 15, 12 and 15 respectively have equal marginals and the others do not. For the two-sample problem there are 53 case studies, 36 of them in two dimensions (19 with equal marginals) and 17 in five dimensions (10 with equal marginals). The discrete cases are discretized versions of the continuous cases in two dimensions, using a 5×5 grid that is the same for all data sets of a case study. Both *MDgof* and *MD2sample* include vignettes that describe all the case studies in detail. Here we will only show a few select examples.

In these examples the difference between a data set generated under the null hypothesis and one generated under the alternative is quite obvious for the purpose of illustration. In the actual case studies this is generally not true, so an actual test is needed.

The alternative distribution always depends on a parameter that the user can choose. In the results described in this paper there are always two of these. One is chosen so that the null hypothesis is actually true (and one can check that the methods achieve the nominal type I error) and a second one is chosen in such a way that for the selected sample size and a true type I error of $\alpha = 0.05$ the best method has a power around $80 - 90\%$. These parameters were chosen with earlier versions of the packages; with the corrected implementations used here the power of the best method is between 75% and 100% in most case studies (median about 93%), and a few have a lower power, see section 10. The sample sizes are 250 for the goodness-of-fit studies and 200 for the two-sample studies.

The comparisons in this paper rest on a working assumption. Say we wish to determine whether method A or method B has a higher power for a specific case (aka $X \sim F$ vs $X \sim G(p)$). We assume that if A has a higher power for one combination of sample size, type I error probability and parameter of the alternative, say $n = 250$, $\alpha = 0.05$ and $p = p_1$, then it also has a higher power for other combinations, such as $n = 100$, $\alpha = 0.01$ and $p = p_2$. In our experience with these and earlier power studies this is usually the case, but power curves can cross, and we have not verified the assumption systematically for the multivariate methods. A check for univariate data supports it: with the R package *R2sample* [Rolke, 2026b], version 5.1.0 (earlier versions give different results), we reran its 100 two-sample case studies (14 methods) with $n = 200$ and $\alpha = 0.1$ instead of $n = 500$ and $\alpha = 0.05$. The mean Kendall rank correlation between the powers of the methods in the two configurations was 0.85, the best method of one configuration was within three percentage points of the best of the other in 95% of the case studies, and the sets of methods within 90% of the best overlapped by 79%. Two replications of the same configuration with different random numbers gave 0.86, 97% and 85%, so the change of sample size and level altered the rankings only slightly more than the Monte Carlo error did (see the replication material). We therefore ran one power study for each case, and the rankings and recommendations of this paper are conditional on the configuration that was studied: the sample sizes given above, $\alpha = 0.05$ and the chosen parameters of the alternatives.

6.1 Computational details

All power studies use $B = 1000$ simulation runs and $\alpha = 0.05$; so the Monte Carlo standard error of a power estimate p is $\sqrt{p(1-p)/1000}$, at most 1.6%. Each case study was run once with the parameter of the alternative and once with the parameter of the null hypothesis, which gives the true type I error of every method in every case study. The type I errors of every method and case study are included in the data sets of the packages.

Table 1 summarizes the true type I errors. The mean type I error of every method over the case studies of a study is between 4.1% and 5.6% (except for the discrete two-sample tests, see below), so no method is liberal. The individual estimates scatter more widely around 5% (between 1.8% and 8.6%) than the binomial error alone would allow. For most methods the power studies find the critical value as the 95% quantile of B simulated values of the test statistic under the null hypothesis, and this quantile has a Monte Carlo error of its own, so the variance of the estimated type I error is about twice the binomial variance. Methods that find their own p values (the chi-square tests, NN0, CF1-CF4 and Ball) do not have this additional error, and their type I errors are indeed closer to 5%. Allowing for this, the number of combinations of method and case study with a type I error above 6.4% (5% plus two binomial standard errors) is about what is expected by chance: 238 against 216 expected over all continuous studies, with no method or case study standing out. The tests themselves, as implemented in *MDgof* and *MD2sample*, are not affected. For discrete data the test statistics can take only finitely many values, so a test cannot achieve the nominal level exactly; the discrete two-sample tests are conservative (means between 3% and 4.5%), so their powers are, if anything, slightly understated relative to the other methods.

Study	Cases	Methods	MeanRange	Min	Max	Above	Expected
Goodness-of-fit, continuous, D=2	60	11	4.8 - 5.2	1.8	8.6	63	45.2
Goodness-of-fit, continuous, D=5	30	5	5.0 - 5.3	2.7	8.2	13	11.8
Goodness-of-fit, discrete	60	8	5.0 - 5.2	2.2	8.1	50	37.8
Hybrid, nMC=1, D=2	30	17	4.4 - 5.6	2.5	8.3	30	28.4
Hybrid, nMC=5, D=2	30	16	4.8 - 5.3	2.6	8.2	37	27.7
Hybrid, nMC=1, D=5	30	15	4.1 - 5.3	2.3	8.2	29	27.0
Hybrid, nMC=5, D=5	30	14	4.7 - 5.3	2.6	8.1	24	26.3
Two-sample, continuous, D=2	36	17	4.5 - 5.3	2.3	7.9	24	34.0
Two-sample, continuous, D=5	17	15	4.6 - 5.3	2.9	8.1	18	15.3
Two-sample, discrete	36	8	3.0 - 4.5	1.3	7.1	3	20.6

Table 1: True type I errors (nominal 5%). Mean range: smallest and largest mean type I error of the methods over the case studies. Min, Max: smallest and largest type I error of any method in any case study. Above: number of method and case study combinations with a type I error above 6.4% (5% plus two standard errors). Expected: the number expected above this value by chance if all methods had a true level of exactly 5%, allowing for the Monte Carlo error of the simulated critical values.

The results were computed with *MDgof* 1.3.0 and *MD2sample* 1.5.0 under R 4.6.0. The versions of all packages used, including the dependencies Rcpp, copula, spatstat, gTests and Ball and *R2sample* 5.1.0, used for the check in section 6, are recorded in the file *renv.lock*, part of the replication material; the environment can be restored with the R package *renv* (`renv::restore()`). Case study k of the two-sample studies uses the seed $2026 + k$, and case study k of family f of the goodness-of-fit studies the seed $2026 + 1000f + k$; the seed is set inside each case study, so the results do not depend on how the case studies are distributed over the cores, and any single result can be reproduced with the routine *run.studies* of the packages. The seeds, the settings and the software versions are stored with the results. Rerunning three case studies with *replication_material.Rmd* on 23 cores reproduced the stored powers exactly. The script that computes all results, *power_studies_rerun.Rmd*, and the script that produces every table and figure of this paper, *replication_material.Rmd*, are part of the replication material of this paper.

Table 2 shows the computing times. Methods that find p values by simulation need the time of one evaluation of the test statistics about $B + 1$ times for one test, the others only once. Most methods are fast, a complete test with $B = 1000$ takes at most a few minutes on one core for all methods of the goodness-of-fit problem and for the methods of the two-sample problem based on distances and nearest neighbors. The exceptions are MMD and Ball divergence for larger data sets.

Problem	Methods	Size	Seconds	Simulation
Goodness-of-fit, continuous, D=2	all	n=250	0.023	yes
Goodness-of-fit, continuous, D=5	all	n=250	0.016	yes
Goodness-of-fit, discrete	all	n=250	0.00043	yes
Two-sample, D=2	KS, K, CvM, AD, NN1, NN5, AZ, BF, BG, MMD	n=200, m=200	0.062	yes
Two-sample, D=2	of these MMD	n=200, m=200	0.053	yes
Two-sample, D=2	FR, NN0, CF1-CF4	n=200, m=200	0.015	no
Two-sample, D=2	Ball	n=200, m=200	0.045	no
Hybrid, nMC=1, D=2	KS, K, CvM, AD, NN1, NN5, AZ, BF, BG, MMD	n=250, m=250	0.09	yes
Hybrid, nMC=1, D=2	of these MMD	n=250, m=250	0.086	yes
Hybrid, nMC=1, D=2	FR, NN0, CF1-CF4	n=250, m=250	0.025	no
Hybrid, nMC=1, D=2	Ball	n=250, m=250	0.096	no
Hybrid, nMC=5, D=2	KS, K, CvM, AD, NN1, NN5, AZ, BF, BG, MMD	n=250, m=1250	0.86	yes
Hybrid, nMC=5, D=2	of these MMD	n=250, m=1250	0.77	yes
Hybrid, nMC=5, D=2	FR, NN0, CF1-CF4	n=250, m=1250	0.32	no
Hybrid, nMC=5, D=2	Ball	n=250, m=1250	6.4	no

Table 2: Computing time in seconds for one evaluation of the test statistics of one data set on one core. For methods that find p values by simulation (last column yes) a test needs this time about B+1 times. For the goodness-of-fit problem the time is for all methods together, including the setup of the test.

For the hybrid tests with an MC data set five times the size of the real data set the Ball divergence test was not run, because with 1500 observations it takes about 6 seconds per data set on a single core (table 2), and about 20 seconds per data set when 23 cores run at the same time, and in all our other studies it was never the only method with good power.

All computations were done on a PC with an Intel Core i9-14900F processor (24 cores) and 32 GB of memory, running Windows 11 and R 4.6.0. The times in table 2 are for a single core. The power studies were run in parallel on 23 cores; rerunning all studies of both packages takes about 23 hours, and about 53 hours if the Ball divergence test is also included in the hybrid studies with nMC=5.

6.2 Naming conventions

We have attempted to name the case studies in a manner that lets the user make an educated guess as to the nature of the study:

6.2.1 Goodness-of-fit

A typical case study is called dist1.dist2, where dist1 is the distribution under the null hypothesis and dist2 is the true distribution of the data. For example, the third study is called normal-ind.normal-cor, indicating that under the null hypothesis the data comes from independent normal distributions whereas the true data comes from correlated normal distributions.

If the case study has distributions that have different marginals the name ends in .marginal.

6.2.2 Two-sample

Here a typical name is dist, indicating that the two data sets come from distribution dist with different parameters. All names end in

- D2: two dimensions, equal marginals
- M: two dimensions, unequal marginals
- D5: five dimensions, equal marginals
- M5: five dimensions, unequal marginals

6.3 Examples

6.3.1 Example 1

The first case study for the goodness-of-fit problem with continuous data, equal marginals, D=2 and no parameter estimation has under the null hypothesis independent uniform distributions in both the x and y variable. Under the alternative hypothesis (or the true data) 30% of the observations are replaced by observations along a diagonal stripe, in such a way that the marginal distributions are still (very nearly) uniform, and so any univariate goodness-of-fit test would be unable to detect any difference and correctly reject the null hypothesis. An example is shown in figure 2 (scatterplots of data sets under the null and under the alternative) and figure 3 (histogram of x under the alternative, showing these are still uniform).

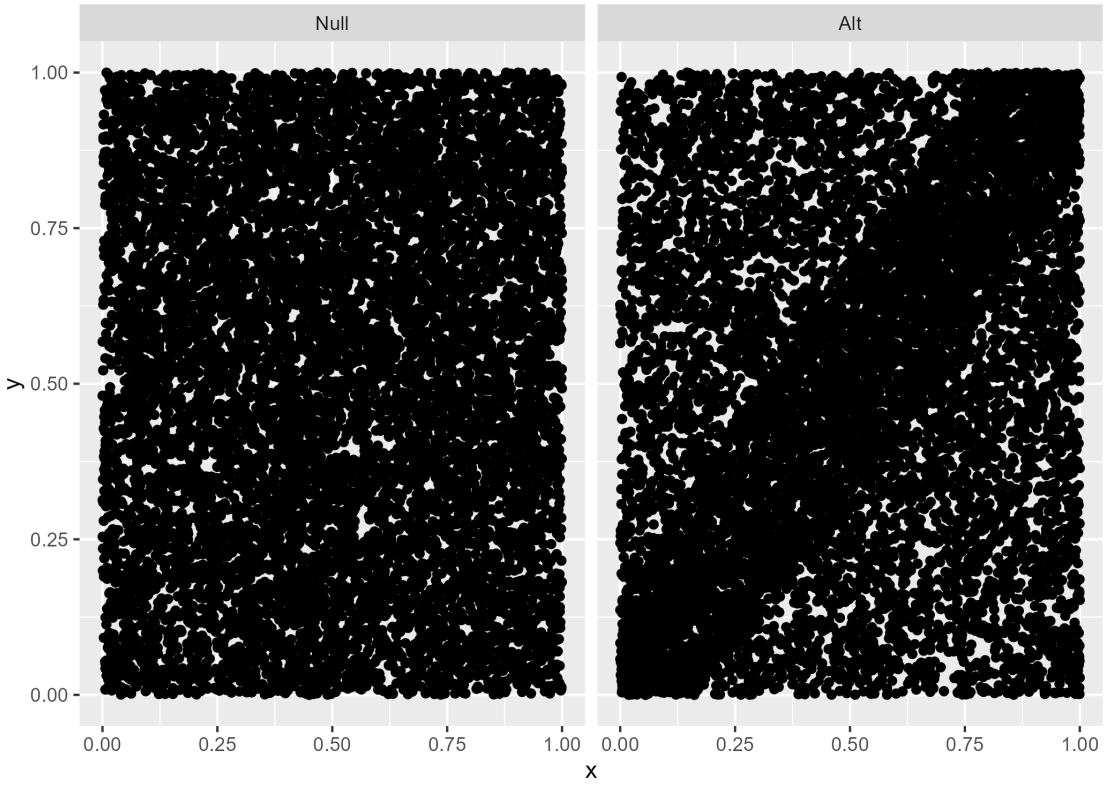

Figure 2: Example 1: data sets under the null hypothesis and under the alternative.

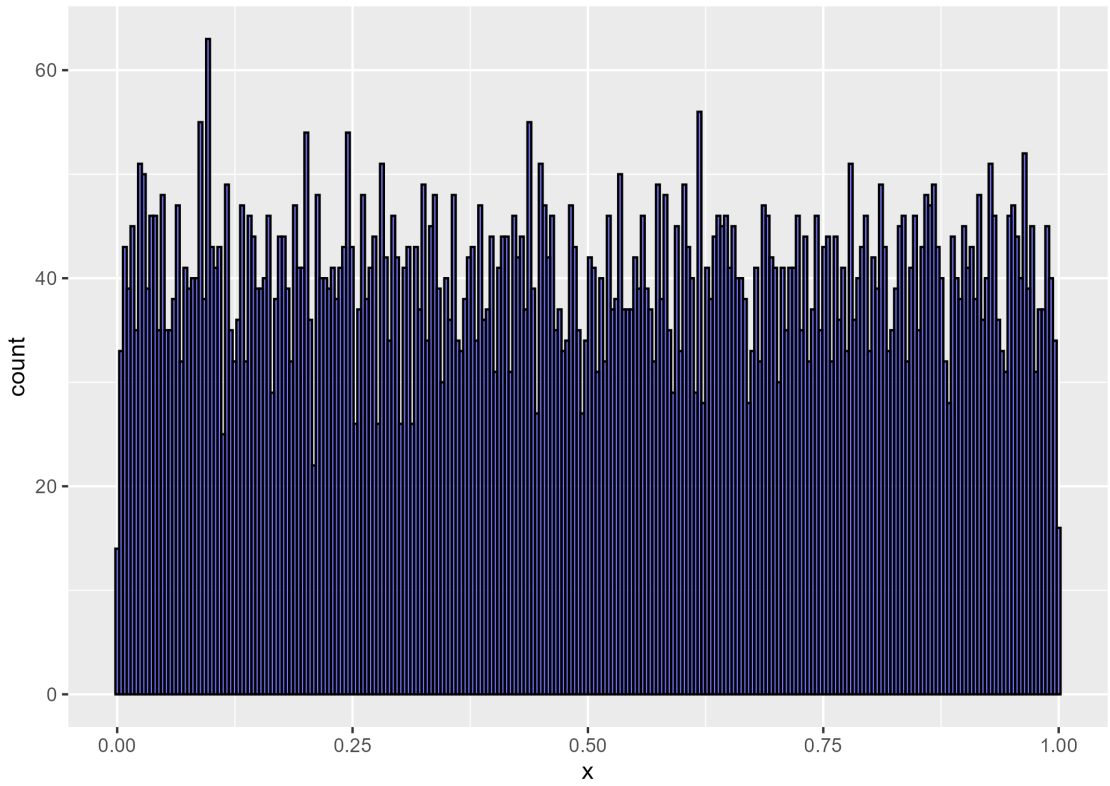

Figure 3: Example 1: histogram of x under the alternative.

The results for this case study are recorded in the data set *power_studies_cont_results*, included in the **R** package *MDgof*. Generally these files are lists with three elements:

1. Null - the powers when the null hypothesis is true
2. Pow - the powers when the null hypothesis is false
3. param_alt - the parameter values under the alternative used

For the first case study we find the powers in table 3.

EP	ES	FF	qAD	qCvM
95	95	57	57	56
KSD	BR	RK	qKS	qK
53	46	42	40	27
BB				
8				

Table 3: Powers of various methods for case study 1 (goodness-of-fit problem), continuous data in two dimensions, no parameter estimation

so the chi-squared tests are best with a power of about 95%, followed by the Fasano-Franceschini test

and the quick Anderson-Darling method with a power of about 57%. Of course in the case of uniform distributions there is no difference between equal-size and equal-probability bins.

6.3.2 Example 2

As an example for discrete data, we use case 15. As with all the case studies for discrete data, these begin with the corresponding continuous case study and then bin the data into 5 by 5 grid. Case study 15 has as the null distribution a 50-50 mixture of Joe and Frank copulas, with the alternative an $(\lambda, 1-\lambda)$ mixture. Table 4 shows a data set under the null hypothesis and one under the alternative, as counts in the 5 by 5 grid. In *MDgof* such a data set is a matrix with three columns, the grid points in x and y and the counts.

Null hypothesis

$y\backslash x$	0.2	0.4	0.6	0.8	1
1	1	2	3	11	50
0.8	2	4	10	23	4
0.6	9	12	20	12	5
0.4	8	22	6	6	1
0.2	24	9	3	3	0

Alternative

$y\backslash x$	0.2	0.4	0.6	0.8	1
1	3	3	3	18	35
0.8	9	4	9	12	11
0.6	3	6	16	10	6
0.4	11	23	13	9	1
0.2	17	17	6	3	2

Table 4: Example 2: counts in the 5 by 5 grid of a data set under the null hypothesis and of a data set under the alternative, case study 15, $n = 250$. The grid points are the upper right corners of the cells.

The results for this case study are recorded in the data set *power_studies_disc_results* of *MDgof*. Table 5 has the powers.

P	TV	KL	H	K	AD	CvM	KS
93	88	83	53	29	26	24	19

Table 5: Powers of various methods for case study 15 (goodness-of-fit problem), discrete data, no parameter estimation

6.3.3 Example 3

As an example of a two-sample problem we use case study 24. This is data from a Dalitz plot, a type of two dimensional data encountered in high energy physics. A Dalitz plot [Dalitz, 1953] describes the decay of a particle into three particles. It shows the squared invariant masses of two pairs of the decay products against each other. If the decay proceeds without any intermediate states, the events are uniformly distributed over the kinematically allowed region, so any structure in the plot, such as a band from an intermediate resonance, is a sign of additional physics. An example is shown in figure 4.

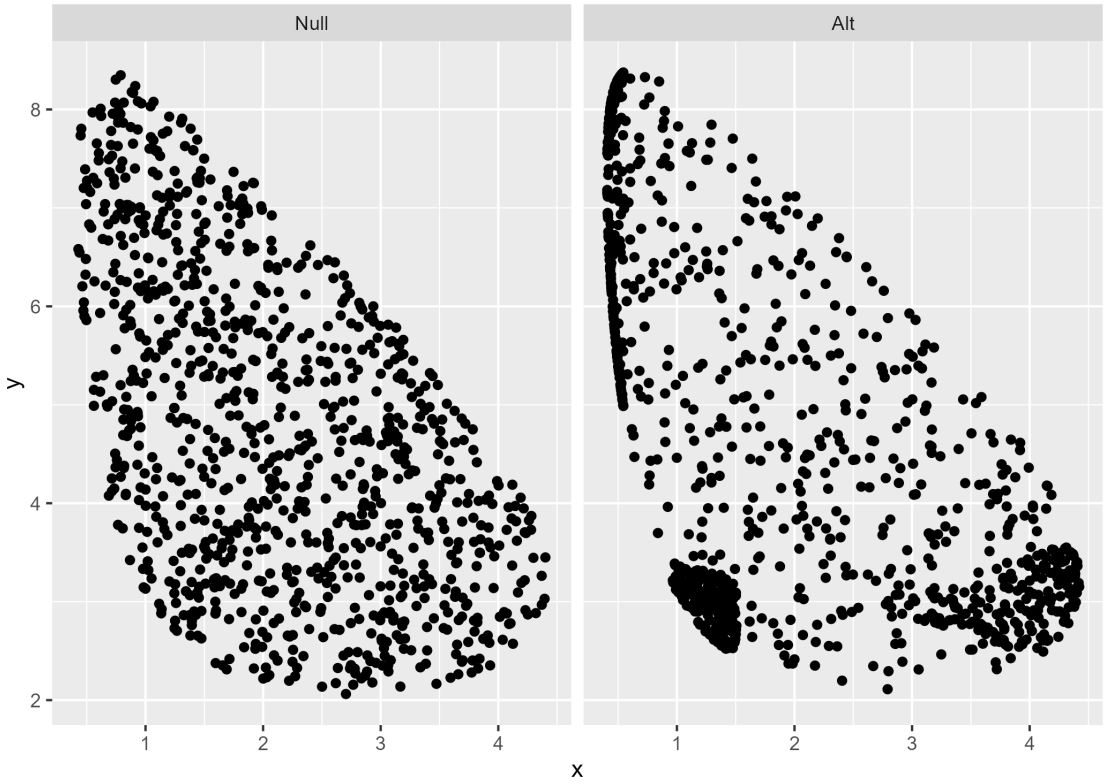

Figure 4: Example 3: data sets under the null hypothesis and under the alternative.

The results of the power study are in table 6. Here the MMD test is best, with a power of 92%, followed by the tests of Aslan and Zech (83%) and of Biswas and Ghosh (82%).

MMD	AZ	BG	NN5	ES	AD	Ball	EP	FR
92	83	82	77	72	66	64	56	52
CF1	CF3	CF4	CvM	NN1	KS	BF	CF2	NN0
52	52	41	37	35	33	31	30	27
K								
26								

Table 6: Powers of various methods for case study 24 (two-sample problem), continuous data

The results of the power studies for the two-sample problem are recorded in the data set *power_studies_results*, part of *MD2sample*. It is a list with six elements:

- *Null Cont*: true type I error for continuous data
- *Alt Cont*: power estimates for continuous data
- *Par Cont*: parameter values for alternative and continuous data
- *Null Disc*: true type I error for discrete data

- *Alt Disc*: power estimates for discrete data
- *Par Disc*: parameter values for alternative and discrete data

7 Results

We will now give the results of the various studies. For each we present the following four tables:

1. Mean power: the mean power of each method over the included case studies.
2. Percentage close to best: the percentage of case studies in which a method had a power within 90% of the best method.
3. Mean shortfall from best: the mean, over the case studies, of the power of the best method minus the power of the method, in percentage points.
4. Best selection(s): for the combination of all case studies we find the smallest selection of methods such that for each study at least one of the methods has a power within 90% of the best. Usually there is more than one such selection.

The first two of these depend on the arbitrary choice of 90%. For the main studies we therefore also show performance profiles [Dolan and Moré, 2002]: for each method the share of case studies in which its power is at least τ times the power of the best method, as a function of τ. A method whose curve is higher is closer to the best in more case studies; the value at $\tau = 0.9$ is the percentage close to best of the tables, the value at $\tau = 1$ the share of case studies in which the method is best.

The best selection rewards a method that is best in a single case study, even if it is weak otherwise. It should therefore be read together with the mean powers and the shortfalls, and in section 8 we check by cross-validation how well the selections carry over to case studies that were not used to find them.

All power estimates are subject to Monte Carlo error. For the methods that find their own p values the standard error of a power estimate is at most 1.6 percentage points with $B = 1000$. For the other methods the critical value is itself estimated from B simulations under the null hypothesis, which adds a further error; for powers between 80% and 90% the standard error is then about 1.5 to 2.5 points, and that of the difference of two power estimates about 3 points. Differences of about three percentage points between two methods in a single case study are therefore not distinguishable, and the same holds for the classification of a method as within 90% of the best when its power is near that threshold. The mean powers are much more precise: over k case studies their standard error is at most about $2.5/\sqrt{k}$ points, below 0.6 points in our studies. The power of the best method is the largest of the estimates of all methods and so is biased upward, by about one percentage point when two or three methods have nearly the same power, which slightly inflates the shortfalls. The best selections are affected most: a method can enter a selection because it is best in one case study by a margin that is within the Monte Carlo error. The general conclusion that no single method can be relied upon does not depend on this, but the exact membership of the selections should be read with this in mind.

In the two-sample problem the Friedman-Rafsky test (FR) is the same test as the original edge-count test of Chen and Friedman (CF1), and the weighted edge-count test (CF3) is the same as CF1 if both data sets have the same size, as is the case in our studies. FR and CF3 are therefore not included in the summaries and the detailed results; the same holds for the hybrid tests with an MC data set of the size of the data, and for FR in the hybrid tests with a larger MC data set.

7.1 Goodness-of-fit, overall results

In this section we include all case studies, whether they have estimated parameters or not and equal marginals or not

7.1.1 Continuous methods - Dim=2

Table 7: Continuous data, D=2, with and without estimation, with and without equal marginals

Mean Power					
ES	EP	FF	RK	qK	qCvM
75	72	69	53	52	48
qKS	qAD	KSD	BR	BB	
47	46	41	37	19	
Percentage close to best					
ES	EP	RK	FF	KSD	qK
50	40	37	30	27	25
qKS	qAD	qCvM	BR	BB	
20	18	17	15	3	
Mean shortfall from best					
ES	EP	FF	RK	qK	qCvM
17	20	23	39	40	44
qKS	qAD	KSD	BR	BB	
45	46	50	55	73	
Best selection(s)					
EP	FF	KSD	qAD	qK	RK
ES	FF	KSD	qAD	qK	RK

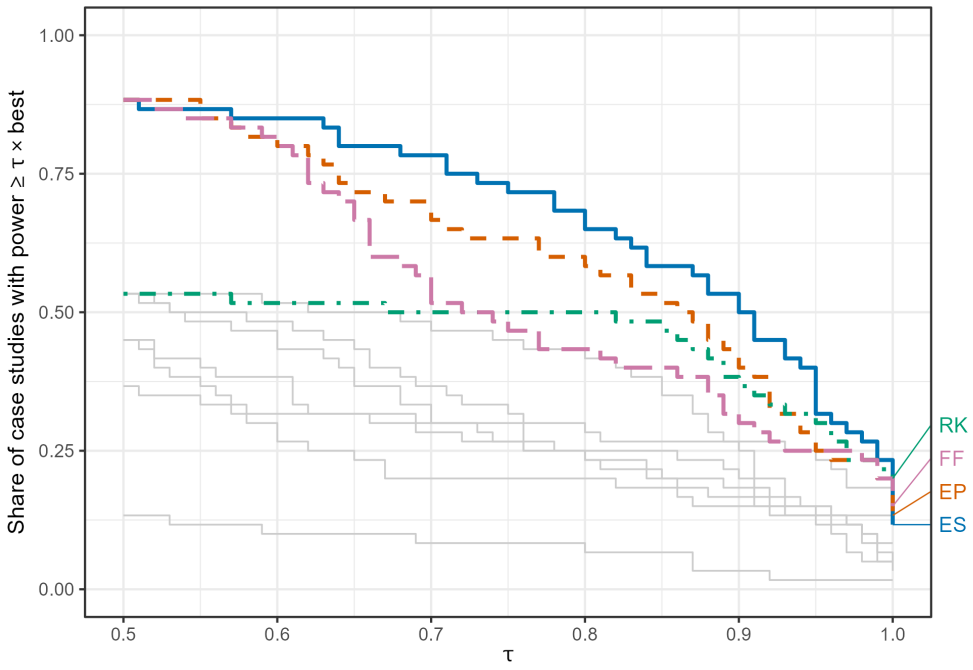

Figure 5: Performance profiles, goodness-of-fit, continuous data, D=2. The four methods with the highest share of case studies within 90% of the best are shown in colour and labelled; all other methods are shown in grey.

The equal-size and equal-probability chi-square tests with a 5 by 5 grid are best, followed by Fasano-Franceschini. There are two best selections with six methods each: FF, KSD, qAD, qK, RK and either ES or EP. In the case studies with parameter estimation the chi-square tests are less dominant, ES and FF have about the same mean power (70% and 68%). Minimum chi-square estimation chooses the parameters that fit the binned data best, and so it can absorb part of a departure from the null hypothesis. An example are the case studies Frank.Frank and Galambos.Galambos, where the alternative adds a diagonal stripe, which on a 5 by 5 grid looks much like a stronger dependence.

7.1.2 Continuous methods - Dim=5

Table 8: Continuous data, D=5, with and without equal marginals

Mean Power				
qKS	qCvM	qK	BR	qAD
68	68	61	39	26
Percentage close to best				
qKS	qK	qCvM	BR	qAD
53	53	53	20	10
Mean shortfall from best				
qKS	qCvM	qK	BR	qAD
15	15	22	44	57
Best selection(s)				
BR	qCvM	qK	qKS	

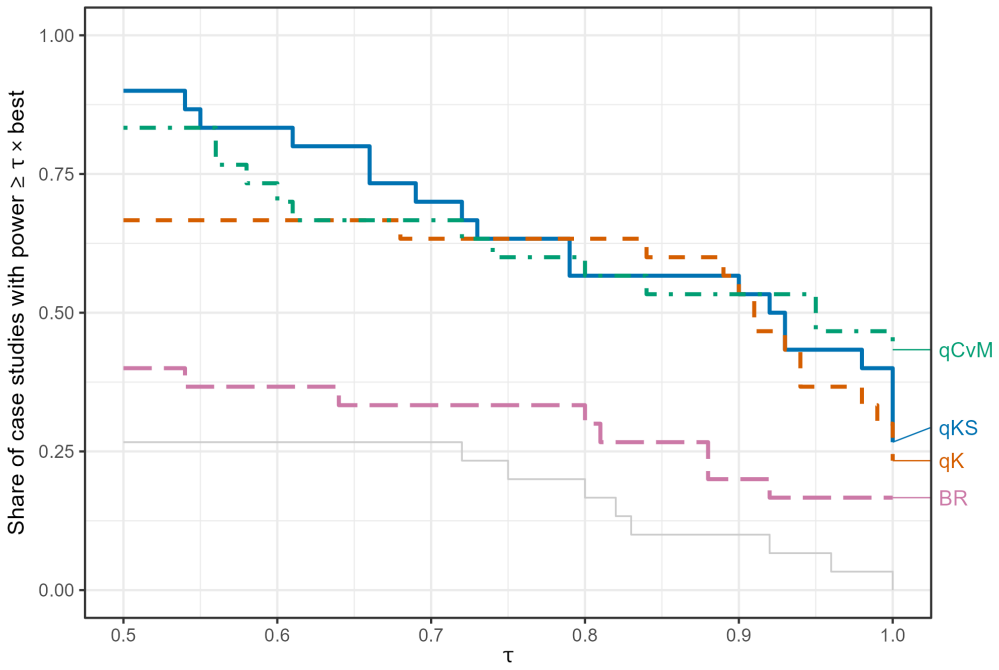

Figure 6: Performance profiles, goodness-of-fit, continuous data, D=5. The four methods with the highest share of case studies within 90% of the best are shown in colour and labelled; all other methods are shown in grey.

7.1.3 Discrete methods

Table 9: Discrete data, with and without estimation, with and without equal marginals

Mean Power			
KL	TV	H	P
81	75	74	71
—	—	—	—
K	AD	CvM	KS
55	54	46	46
Percentage close to best			
KL	P	H	TV
72	55	53	47
—	—	—	—
K	AD	KS	CvM
40	27	22	18
Mean shortfall from best			
KL	TV	H	P
8	14	15	18
—	—	—	—
K	AD	CvM	KS
34	35	43	43
Best selection(s)				
CvM	H	K	KS	P
CvM	K	KL	KS	P
AD	H	K	KS	P
AD	K	KL	KS	P

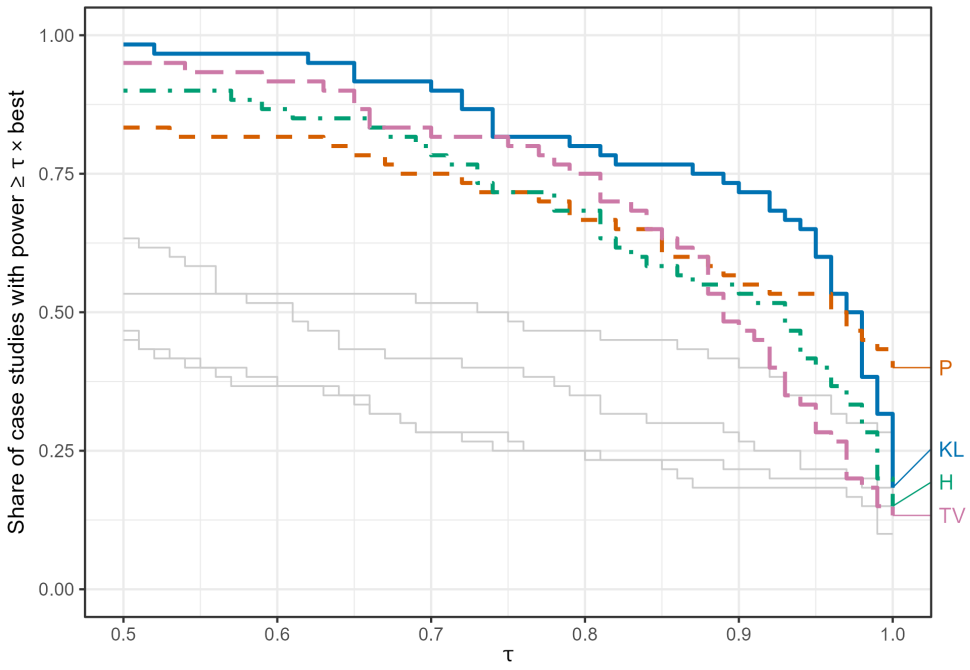

Figure 7: Performance profiles, goodness-of-fit, discrete data. The four methods with the highest share of case studies within 90% of the best are shown in colour and labelled; all other methods are shown in grey.

7.1.4 Continuous plus Hybrid methods

To distinguish between methods that exist for both the goodness-of-fit and two-sample problem the abbreviations of the first end in G and the second in T1 if the MC sample size is equal to the one of the data set and T5 if it is five times as large.

Table 10: Power of gof vs hybrid methods, Dim=2, methods with at least 50 mean power only

Mean Power

ESG	EPG	MMDT5	EST5	FFG	EPT5	ADT5	qCvMG	qADG
81	77	77	72	70	68	57	55	53
MMDT1	qKG	BFT5	qKSG	KSDG				
52	52	51	51	51				

Percentage close to best

ESG	EPG	MMDT5	KSDG	EST5	FFG	BFT5	qKSG	EPT5
67	53	53	33	30	27	27	23	23
qCvMG	qKG	qADG	ADT5	MMDT1				
20	17	17	10	7				

Mean shortfall from best

ESG	EPG	MMDT5	EST5	FFG	EPT5	ADT5	qCvMG	qADG
11	14	15	19	21	23	35	37	38
MMDT1	qKG	BFT5	qKSG	KSDG				
39	39	40	40	41				

Best selection(s)

BFT5	ESG	KSDG	qKG
ESG	FFG	KSDG	qKG

Table 11: Power of gof vs hybrid methods, Dim=5, methods with at least 50 mean power only

Mean Power								
MMDT5	AZT5	MMDT1	qKSG	qCvMG	CvMT5	KST5	AZT1	qKG
85	78	72	68	68	67	65	61	61
ADT5	BFT5	NN5T5	KT5	BallT1				
58	57	57	57	55				
Percentage close to best								
MMDT5	AZT5	qCvMG	BallT1	MMDT1	BFT5	ADT5	AZT1	KST5
80	63	47	43	40	40	37	33	30
KT5	CvMT5	qKSG	qKG	NN5T5				
30	30	20	20	10				
Mean shortfall from best								
MMDT5	AZT5	MMDT1	qKSG	qCvMG	CvMT5	KST5	AZT1	qKG
9	17	22	26	27	27	29	33	33
ADT5	BFT5	NN5T5	KT5	BallT1				
36	37	37	37	39				
Best selection(s)								
BallT1	MMDT5	qCvMG						

7.1.5 Continuous methods, with or without equal marginals

Here we distinguish between case studies with marginals that are equal under the null and the alternative (so that univariate methods would not detect a difference) and those with different marginals.

1. With equal marginals

Table 12: Continuous data, with and without estimation, with equal marginals

Mean Power					
EP	ES	FF	RK	qAD	qCvM
79	78	66	53	40	37
KSD	qKS	BR	qK	BB	
30	30	27	26	26	
Percentage close to best					
EP	ES	RK	FF	qAD	KSD
70	67	40	17	10	10
qKS	qK	qCvM	BB	BR	
7	7	7	7	0	
Mean shortfall from best					
EP	ES	FF	RK	qAD	qCvM
12	13	25	38	51	54
KSD	qKS	BR	qK	BB	
61	61	64	65	65	
Best selection(s)					
EP	qAD	qK	RK		
EP	qAD	qKS	RK		
EP	qAD	qCvM	RK		
EP	FF	qAD	RK		

2. Without equal marginals

Table 13: Continuous data, with and without estimation, without equal marginals

Mean Power					
qK	FF	ES	qKS	EP	qCvM
79	73	72	65	65	60
KSD	RK	qAD	BR	BB	
53	52	51	48	13	
Percentage close to best					
qK	KSD	FF	qKS	RK	ES
43	43	43	33	33	33
BR	qCvM	qAD	EP	BB	
30	27	27	10	0	
Mean shortfall from best					
qK	FF	ES	qKS	EP	qCvM
14	20	21	28	29	33
KSD	RK	qAD	BR	BB	
40	41	42	45	80	
Best selection(s)					
FF	KSD	qAD	qK	RK	
ES	FF	KSD	qAD	qK	

There is a remarkable difference between these two types of case studies. In the case of equal marginals the chi-square tests are much better than any others, with a mean power of 79% for EP and 78% for ES, and the best selections consist of EP, qAD and RK plus one more method. In the case of unequal marginals the quick Kuiper test has the highest mean power, the two best selections need five methods each (FF, KSD, qAD, qK and either ES or RK) and do not include EP.

In practice the alternative is unknown, and so is whether the marginals are correct. Choosing the methods after a preliminary test of the marginals on the same data would change the level of the final test, and we do not recommend it. Instead, the overall selections (section 4 and tables 7 and 8) should be used, as they were found from both kinds of case studies. If there is prior reason to believe that the marginals are correct, for example when the marginals are well established and the question is about the dependence structure, as in copula modelling, a chi-square test with a small number of bins should be included.

7.2 Two-sample

7.2.1 Continuous methods - overall results

Table 14: Continuous data, D=2, with and without equal marginals

Mean Power									
MMD	AZ	ES	NN5	EP	CF1	AD	BF	K	CvM
89	84	80	79	76	58	58	56	53	51
CF4	Ball	KS	NN1	CF2	NN0	BG			
51	51	50	47	43	37	34			
Percentage close to best									
MMD	AZ	ES	NN5	EP	BF	Ball	K	KS	AD
86	69	58	50	47	28	19	17	14	14
CvM	BG	CF1	CF4	NN1	NN0	CF2			
11	11	6	6	3	3	3			
Mean shortfall from best									
MMD	AZ	ES	NN5	EP	CF1	AD	BF	K	CvM
5	10	13	14	17	35	35	38	41	42
CF4	Ball	KS	NN1	CF2	NN0	BG			
42	43	44	46	51	57	59			
Best selection(s)									
BG	ES	MMD							

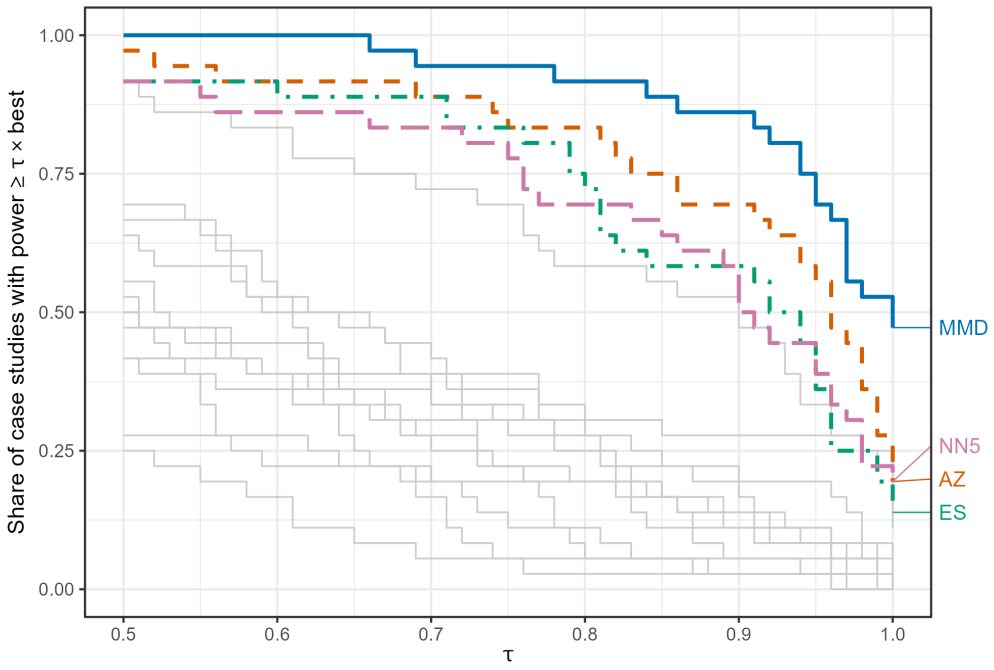

Figure 8: Performance profiles, two-sample problem, continuous data, D=2. The four methods with the highest share of case studies within 90% of the best are shown in colour and labelled; all other methods are shown in grey.

Table 15: Continuous data, D=5, with and without equal marginals

Mean Power

MMD	NN5	AZ	AD	CvM	CF4	KS	CF2	CF1
82	69	69	63	61	52	51	49	49
Ball	BF	NN1	K	BG	NN0			
48	47	42	39	34	25			

Percentage close to best

MMD	AZ	CvM	AD	BF	NN5	BG	CF2	CF4
65	53	47	35	29	24	18	18	18
Ball	KS	NN1	CF1	K	NN0			
18	12	12	12	6	6			

Mean shortfall from best

MMD	NN5	AZ	AD	CvM	CF4	KS	CF2	CF1
9	22	22	28	30	39	40	42	42
Ball	BF	NN1	K	BG	NN0			
44	45	49	52	57	66			

Best selection(s)

BF	BG	CvM	MMD	NN1
AD	BF	BG	MMD	NN1
BF	BG	CF1	CvM	MMD
BF	BG	CF2	CvM	MMD
AD	BF	BG	CF1	MMD
AD	BF	BG	CF2	MMD
BF	BG	CF4	CvM	MMD
AD	BF	BG	CF4	MMD
AZ	BG	CvM	MMD	NN1
AD	AZ	BG	MMD	NN1
BF	BG	CvM	MMD	NN5
AD	BF	BG	MMD	NN5
AZ	BG	CF1	CvM	MMD
AZ	BG	CF2	CvM	MMD
AD	AZ	BG	CF1	MMD
AD	AZ	BG	CF2	MMD
AZ	BG	CF4	CvM	MMD
AD	AZ	BG	CF4	MMD
AZ	BG	CvM	MMD	NN5
AD	AZ	BG	MMD	NN5

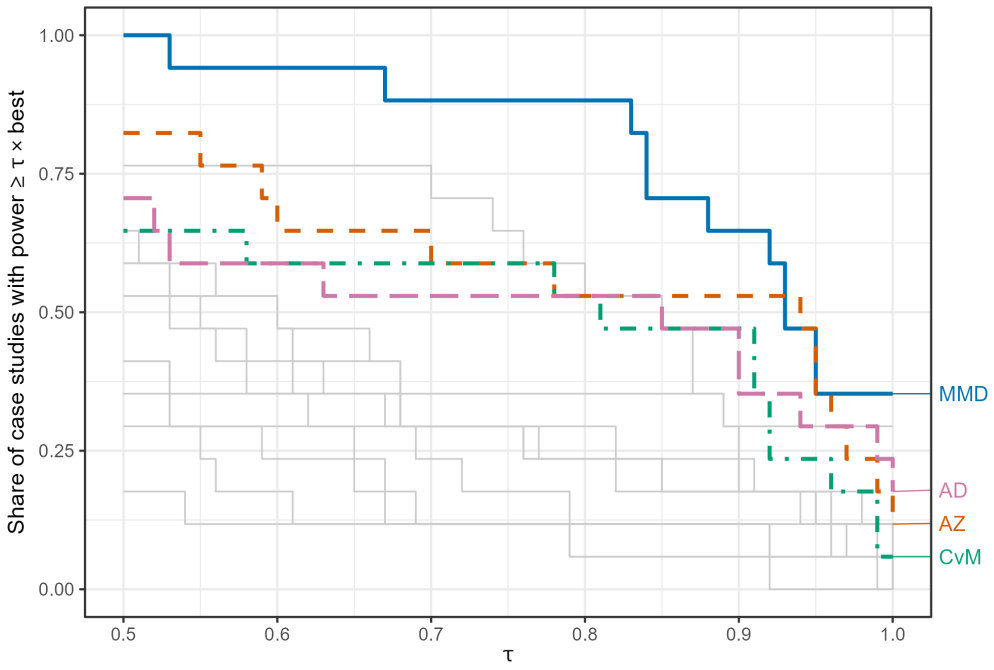

Figure 9: Performance profiles, two-sample problem, continuous data, D=5. The four methods with the highest share of case studies within 90% of the best are shown in colour and labelled; all other methods are shown in grey.

In both 2 and 5 dimensions MMD is best, followed by AZ (and in 5 dimensions NN5). The best selections contain BG because it is the best method in a single case study (NormalStretchM), although on average it is the weakest method in two dimensions. This illustrates that the best selection should not be read on its own but together with the mean powers.

7.2.2 Continuous methods - with or without equal marginals

1. With equal marginals

Table 16: Continuous data, with equal marginals

Mean Power									
MMD	ES	NN5	AZ	EP	CF1	CF4	AD	NN1	CvM
90	86	85	83	83	64	57	55	53	52
BF	CF2	KS	Ball	NN0	K	BG			
50	49	43	42	40	38	17			
Percentage close to best									
MMD	ES	AZ	EP	NN5	BF	Ball	CvM	AD	CF1
89	74	68	68	58	16	16	11	11	11
CF4	KS	K	NN1	BG	NN0	CF2			
11	5	5	5	5	5	5			
Mean shortfall from best									
MMD	ES	NN5	AZ	EP	CF1	CF4	AD	NN1	CvM
3	7	9	10	11	30	37	38	41	42
BF	CF2	KS	Ball	NN0	K	BG			
44	44	51	52	54	55	77			
Best selection(s)									
EP	NN5								
ES	NN5								
EP	MMD								
ES	MMD								

2. Without equal marginals

Table 17: Continuous data, without equal marginals

Mean Power									
MMD	AZ	ES	NN5	EP	K	BF	AD	Ball	KS
87	84	73	72	70	69	62	61	61	57
BG	CF1	CvM	CF4	NN1	CF2	NN0			
54	51	50	44	41	35	34			
Percentage close to best									
MMD	AZ	NN5	BF	ES	K	KS	Ball	EP	AD
82	71	41	41	41	29	24	24	24	18
BG	CvM	NN1	NN0	CF1	CF2	CF4			
18	12	0	0	0	0	0			
Mean shortfall from best									
MMD	AZ	ES	NN5	EP	K	BF	AD	Ball	KS
6	9	20	21	24	24	31	32	32	36
BG	CF1	CvM	CF4	NN1	CF2	NN0			
39	42	43	49	52	58	59			
Best selection(s)									
BG	ES	MMD							

7.2.3 Discrete methods - overall results

Table 18: Discrete data, with and without equal marginals

Mean Power							
Chisquare	BF	NN	AD	K	CvM	KS	AZ
81	55	55	49	48	44	43	41
Percentage close to best							
Chisquare	BF	AZ	K	AD	NN	KS	CvM
83	28	19	17	14	11	8	8
Mean shortfall from best							
Chisquare	BF	NN	AD	K	CvM	KS	AZ
4	30	30	35	37	41	42	43
Best selection(s)							
AZ	Chisquare	K					
BF	Chisquare	K					

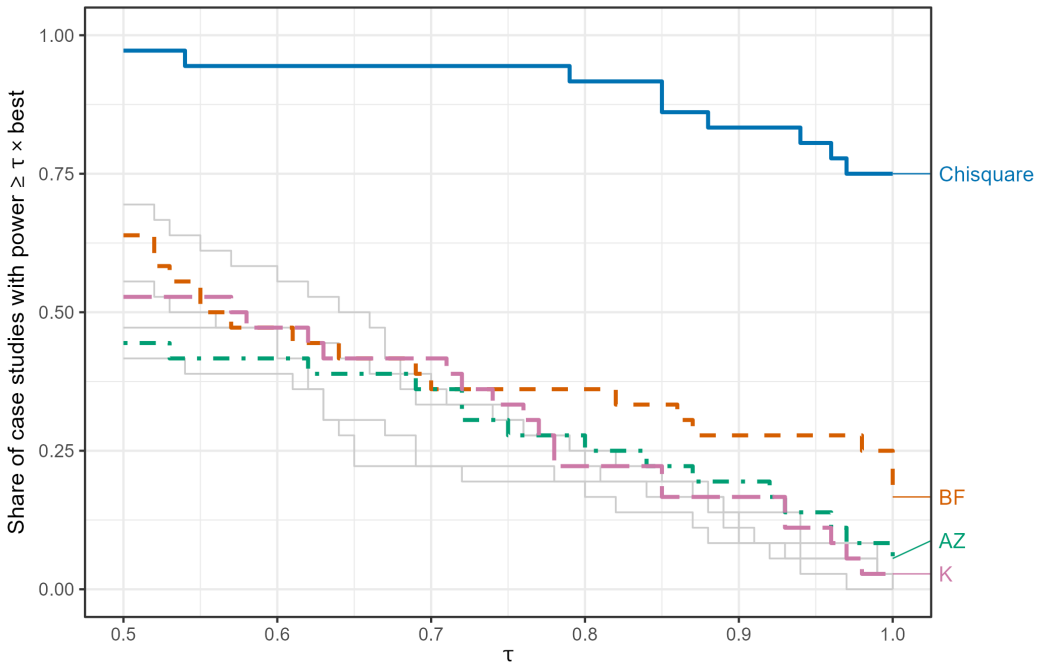

Figure 10: Performance profiles, two-sample problem, discrete data. The four methods with the highest share of case studies within 90% of the best are shown in colour and labelled; all other methods are shown in grey.

7.2.4 Discrete methods - with or without equal marginals

1. With equal marginals

Table 19: Discrete data, with equal marginals

Mean Power							
Chisquare	NN	AD	BF	CvM	KS	K	AZ
87	61	46	45	43	35	32	23
Percentage close to best							
Chisquare	NN	BF	AD	KS	K	CvM	AZ
100	16	16	11	5	5	5	5
Mean shortfall from best							
Chisquare	NN	AD	BF	CvM	KS	K	AZ
0	27	41	43	45	52	55	64
Best selection(s)							
AZ	Chisquare						
Chisquare	K						
Chisquare	KS						
Chisquare	CvM						
BF	Chisquare						
AD	Chisquare						
Chisquare	NN						

2. Without equal marginals

Table 20: Discrete data, without equal marginals

Mean Power							
Chisquare	BF	K	AZ	AD	KS	NN	CvM
73	66	65	62	53	52	48	45
Percentage close to best							
Chisquare	BF	AZ	K	AD	KS	CvM	NN
65	41	35	29	18	12	12	6
Mean shortfall from best							
Chisquare	BF	K	AZ	AD	KS	NN	CvM
9	16	17	20	29	30	34	37
Best selection(s)							
AZ	Chisquare	K					
BF	Chisquare	K					

8 Cross-validation of the best selections

The best selections were found from the same case studies on which they are judged. To see how well they carry over to new problems we use leave-one-out cross-validation: for each case study we find the best selection from all the other case studies of the same study and then check it on the case study that was left

out. This needs no new simulations, only the stored powers. Table 21 has the results. For comparison it also shows the regret of the same number of methods with the highest mean power, and of the single method with the highest mean power.

Study	Cases	Selection (all cases)	Same	Covered	Regret	Top-k	Single	Worst
Goodness-of-fit, continuous, D=2	60	qK, qAD, KSD, FF, RK, ES	98	98	0.7	2.7	16.9	17
Goodness-of-fit, continuous, D=5	30	qKS, qK, qCvM, BR	97	97	0.2	0.0	21.9	5
Goodness-of-fit, discrete	60	KS, K, AD, KL, P	97	97	0.6	1.9	8.1	10
Two-sample, continuous, D=2	36	BG, MMD, ES	97	97	2.2	2.1	4.6	25
Two-sample, continuous, D=5	17	AD, NN5, AZ, BG, MMD	82	82	3.7	2.8	9.4	33
Two-sample, discrete	36	K, BF, Chisquare	97	97	0.5	0.5	4.2	10

Table 21: Leave-one-out cross-validation of the best selections. Same: percentage of left out case studies for which the selection found from the other case studies is the same as the one from all case studies. Covered: percentage of left out case studies in which the selection contains a method within 90% of the best. Regret: mean of the power of the best method minus the power of the best selected method, in percentage points. Top-k: the same for the same number of methods with the highest mean power. Single: the same for the single method with the highest mean power. Worst: largest regret of the selection.

The selections are quite stable: in five of the six studies the selection found without one case study is the same as the one found from all case studies in at least 97% of the cases, and the left out case study is covered (a selected method has a power within 90% of the best) equally often. The mean loss of power by using the selection instead of the best method is less than one percentage point for the goodness-of-fit problems and the discrete two-sample problem, and about two to four percentage points for the continuous two-sample problems. Using a single method, even the one with the highest mean power, loses between four and twenty-two percentage points on average, which confirms that no single method can be recommended.

The cross-validation also shows the weak point of the best selection. The case studies that are not covered are those in which a single method is far better than all others, for example NormalStretchM, where only BG has good power, and FrankNormalM5 in five dimensions. A method that wins only such a case study enters the selection, but for a new problem it helps only if that problem is of the same kind. In five dimensions, with only 17 case studies, the selection is least stable, and there the methods with the highest mean power do somewhat better than the best selection. Note also that our case studies are not independent, several of them use the same copula with different marginals, and so leave-one-out cross-validation probably somewhat overstates how well the selections carry over.

Our recommendations in section 4 therefore combine the two constructions. We start from the best selection, but a method that enters it only because it is best in a single case study and is weak on average (such as BG in the two-sample problem) is replaced by the method with the next highest mean power. Where there are few case studies, as in five dimensions, the selection is less stable, and there we recommend the methods with the highest mean power (for the goodness-of-fit problem these coincide with the best selection).

9 Continuous vs Discrete Methods

In many of the case studies we ran both continuous and discrete methods on the same data sets, after discretizing the continuous data set. This allows us to compare methods from both types of data. The result

is shown in figure 11. We see that, as one would expect, mostly discretizing comes at the cost of some power. However, there are some exceptions, especially on the goodness-of-fit problems.



Figure 11: Power (in %) of the best method for continuous data against the power of the best method for the discretized data, one point per case study. The line is the line of equality; points below it are case studies where discretizing costs power.

10 Limitations

The conclusions of this paper are based on a collection of case studies chosen by the author, who is also the author of the two R packages that implement the methods. We have tried to include a wide variety of problems, but other kinds of alternatives might well favor other methods; the cross-validation in section 8 checks the selections only within our collection. All studies use a single sample size (250 for the goodness-of-fit and 200 for the two-sample problem), $\alpha = 0.05$, two and five dimensions and, for discrete data, a 5×5 grid. The parameter of each alternative was chosen so that the best method has a power of about 80% to 90%, which makes the mean powers depend on this choice. In particular, the rankings of the methods were found for a single configuration of each case study. That they carry over to other sample sizes, levels and alternatives is a working assumption (see section 6), not a result of this paper.

The studies also revealed some weaknesses of individual methods. In five dimensions the quick Anderson-Darling test qAD has a power below the nominal level for several alternatives, so it is biased for them, presumably because of the weight $1/(F(1-F))$, which becomes very large near the corners of the unit cube. For discrete data the Aslan-Zech test is biased for the two case studies with non-monotone dependence (CheckerboardD2 and TentD2). The discrete two-sample tests are somewhat conservative (see section 6.1). Finally, in four of the case studies with parameter estimation for discrete data all methods have a power above 95%, and in a few case studies in five dimensions even the best method has a power below 50%, so these case studies contribute little to the comparison.

11 Detailed results

In this section we show the detailed results of all the power studies. These are presented as follows. For each case study the methods are sorted according to their power, with the method with the highest power first. In the tables these are shown in dark red. Next come any method whose power is at least 90% of the highest, and these are shown in light red. Any method with a power at least 75% of best is in green and within 50% in blue. Any method with a power less than 50% of best is shown without color. Methods whose powers differ by less than about three percentage points should be regarded as tied; their order, and the boundaries between the color classes, are subject to Monte Carlo error.

The actual powers, as well as the actual type I errors, are included as data sets in the R packages. As in the summaries, FR and CF3 are not shown in the two-sample tables.

11.1 Goodness-of-fit

11.1.1 Continuous data, no estimation, D=2

uniform.uniform-diagonal-n	EP	ES	FF	qAD	qCvM	KSD
uniform.uniform-diagonal-b	EP	ES	qAD	qCvM	BR	FF
normal-ind.normal-cor	KSD	ES	EP	qAD	qCvM	FF
t-ind.t-cor	EP	KSD	ES	qAD	qCvM	FF
uniform.Frank	ES	EP	qAD	FF	qCvM	BR
uniform.Clayton	ES	EP	qAD	qCvM	FF	BR
uniform.Gumbel	ES	EP	qAD	FF	qCvM	BR
uniform.Galambos	ES	EP	qCvM	qAD	FF	BR
uniform.HuslerReiss	EP	ES	qAD	FF	qCvM	KSD
uniform.Joe	EP	ES	FF	qCvM	qAD	KSD
mix.Clayton.Gumbel	ES	EP	RK	FF	BR	qK
mix.uniform.Frank	ES	EP	RK	FF	KSD	qCvM
mix.Clayton.Frank	RK	ES	EP	FF	BB	qK
mix.Frank.Gumbel	RK	EP	ES	FF	BR	qK
mix.Frank.Joe	RK	ES	EP	FF	qK	BR
normal.normal-shift-one.marginal	FF	KSD	qKS	qCvM	ES	qK
normal.normal-shift-both.marginal	qCvM	KSD	qKS	FF	qAD	ES
normal.normal-stretch-one.marginal	KSD	qAD	RK	qK	ES	FF
normal.normal-stretch-both.marginal	KSD	RK	ES	qAD	qK	FF
normal.stretch-shift-one.marginal	KSD	FF	qKS	ES	qK	qCvM
normal.stretch-shift-two.marginal	qKS	qCvM	KSD	FF	ES	qAD
uniform.rotated-uniform.marginal	FF	BR	qK	qAD	qKS	qCvM
uniform.beta-one.marginal	qCvM	qKS	FF	qAD	BR	qK
uniform.beta-two.marginal	KSD	BR	RK	qK	FF	EP
uni-exp-1.uni-exp-l.marginal	KSD	FF	qKS	qCvM	qAD	BR
exp-exp-1.exp-exp-l.marginal	FF	qKS	qCvM	ES	EP	qAD
beta-nor-1.beta-nor-mean.marginal	qK	ES	RK	EP	FF	qKS
beta-nor-1.beta-nor-sd.marginal	qK	ES	RK	EP	qCvM	FF
beta-beta-2.beta-beta-a.marginal	qK	RK	BR	ES	EP	FF
beta05.normal.marginal	RK	ES	BR	qK	EP	FF

uniform.uniform-diagonal-n	BR	RK	qKS	qK	BB
uniform.uniform-diagonal-b	KSD	qKS	qK	RK	BB
normal-ind.normal-cor	qKS	qK	BB	RK	BR
t-ind.t-cor	qKS	qK	BB	BR	RK
uniform.Frank	KSD	qKS	qK	RK	BB
uniform.Clayton	KSD	qKS	qK	RK	BB
uniform.Gumbel	KSD	qKS	qK	RK	BB
uniform.Galambos	KSD	qKS	qK	BB	RK
uniform.HuslerReiss	BR	qKS	qK	RK	BB
uniform.Joe	BR	qKS	qK	RK	BB
mix.Clayton.Gumbel	qKS	qCvM	qAD	BB	KSD
mix.uniform.Frank	qAD	BR	qKS	qK	BB

mix.Clayton.Frank	qKS	BR	qCvM	qAD	KSD
mix.Frank.Gumbel	qKS	qCvM	BB	qAD	KSD
mix.Frank.Joe	qKS	qCvM	BB	qAD	KSD
normal.normal-shift-one.marginal	EP	qAD	BB	RK	BR
normal.normal-shift-both.marginal	EP	qK	BR	BB	RK
normal.normal-stretch-one.marginal	EP	qKS	qCvM	BR	BB
normal.normal-stretch-both.marginal	EP	qKS	qCvM	BR	BB
normal.stretch-shift-one.marginal	EP	qAD	RK	BR	BB
normal.stretch-shift-two.marginal	qK	EP	RK	BR	BB
uniform.rotated-uniform.marginal	EP	ES	RK	KSD	BB
uniform.beta-one.marginal	KSD	EP	ES	RK	BB
uniform.beta-two.marginal	ES	qKS	qCvM	qAD	BB
uni-exp-1.uni-exp-l.marginal	ES	EP	qK	RK	BB
exp-exp-1.exp-exp-l.marginal	qK	KSD	RK	BB	BR
beta-nor-1.beta-nor-mean.marginal	qCvM	BR	BB	qAD	KSD
beta-nor-1.beta-nor-sd.marginal	qKS	BR	BB	qAD	KSD
beta-beta-2.beta-beta-a.marginal	qKS	qCvM	BB	qAD	KSD
beta05.normal.marginal	qAD	qKS	qCvM	BB	KSD

Table 22: Power, continuous data, D=2, no estimation

11.1.2 Continuous data, with estimation, D=2

normal-sigma.t	qAD	ES	KSD	RK	FF	qK
Frank.Frank	FF	RK	BB	ES	EP	KSD
Clayton.Clayton	BB	EP	ES	RK	FF	qK
Gumbel.Gumbel	RK	EP	ES	FF	BB	qK
Galambos.Galambos	RK	FF	BB	ES	EP	qK
HuslerReiss.HuslerReiss	RK	EP	ES	FF	BB	qK
Joe.Joe	RK	EP	ES	BB	FF	qAD
mix.uniform.Frank	KSD	EP	ES	RK	FF	qAD
mix.Clayton.Frank	RK	EP	ES	FF	qCvM	qKS
mix.Clayton.Clayton	FF	RK	BB	ES	EP	qK
mix.uniform.Joe	RK	ES	EP	BB	FF	qAD
mix.uniform.Plackett	EP	ES	KSD	FF	BR	qCvM
Clayton.marginal.normal-sigma	FF	EP	ES	qK	BR	qKS
Gumbel.marginal.normal-sigma	qCvM	qKS	qAD	FF	qK	ES
Joe.marginal.exponential	qCvM	FF	qKS	qK	RK	qAD
normal-mean.t.marginal	qAD	ES	KSD	RK	FF	BR
Clayton.marginal.normal.marginal	RK	qK	qAD	FF	ES	qCvM
Joe.marginal.exponential.marginal	KSD	qCvM	qKS	qAD	FF	qK
Plackett.marginal.beta22.marginal	BR	RK	KSD	qK	FF	ES
Frank.marginal.dblexp.marginal	KSD	RK	qK	FF	EP	ES
Frank.marginal.linear.marginal	BR	FF	qK	qKS	qCvM	ES
Joe.marginal.truncexp.marginal	FF	BR	qK	qKS	ES	EP
Clayton.marginal.betaa1.marginal	FF	ES	BR	EP	qK	qKS
beta22.normal-mean.marginal	qK	ES	EP	RK	qCvM	KSD
beta22.normal-sd.marginal	qK	RK	ES	EP	FF	qKS
beta22.lognormal-mean.marginal	BR	KSD	qK	ES	RK	EP
beta22.lognormal-sd.marginal	ES	RK	qK	EP	FF	qKS
beta22.exponential.marginal	qK	ES	RK	qCvM	EP	qKS
exp1.normal-sd.marginal	qKS	qK	qCvM	qAD	FF	EP
beta05.gamma.marginal	qK	RK	KSD	FF	qAD	qKS
normal-sigma.t	BR	EP	qCvM	qKS	BB	
Frank.Frank	qK	qKS	BR	qCvM	qAD	
Clayton.Clayton	qCvM	qKS	qAD	BR	KSD	
Gumbel.Gumbel	qCvM	qKS	BR	qAD	KSD	
Galambos.Galambos	qCvM	qAD	qKS	BR	KSD	
HuslerReiss.HuslerReiss	qCvM	qKS	qAD	BR	KSD	
Joe.Joe	qK	qCvM	qKS	BR	KSD	
mix.uniform.Frank	qCvM	qKS	BR	qK	BB	
mix.Clayton.Frank	qAD	qK	BB	BR	KSD	
mix.Clayton.Clayton	qKS	qCvM	qAD	BR	KSD	

mix.uniform.Joe	qK	qCvM	qKS	BR	KSD
mix.uniform.Plackett	qK	qKS	qAD	RK	BB
Clayton.marginal.normal-sigma	qCvM	qAD	RK	BB	KSD
Gumbel.marginal.normal-sigma	EP	KSD	RK	BR	BB
Joe.marginal.exponential	EP	ES	BB	BR	KSD
normal-mean.t.marginal	qK	qKS	EP	qCvM	BB
Clayton.marginal.normal.marginal	qKS	EP	BR	BB	KSD
Joe.marginal.exponential.marginal	ES	EP	RK	BR	BB
Plackett.marginal.beta22.marginal	EP	qKS	BB	qCvM	qAD
Frank.marginal.dblexp.marginal	qKS	BR	qCvM	qAD	BB
Frank.marginal.linear.marginal	EP	KSD	qAD	RK	BB
Joe.marginal.truncexp.marginal	qCvM	qAD	RK	BB	KSD
Clayton.marginal.betaa1.marginal	qCvM	qAD	BB	RK	KSD
beta22.normal-mean.marginal	qKS	FF	BR	BB	qAD
beta22.normal-sd.marginal	qCvM	qAD	BB	BR	KSD
beta22.lognormal-mean.marginal	qCvM	qKS	FF	BB	qAD
beta22.lognormal-sd.marginal	qCvM	qAD	BB	BR	KSD
beta22.exponential.marginal	FF	BR	KSD	qAD	BB
exp1.normal-sd.marginal	ES	BR	KSD	RK	BB
beta05.gamma.marginal	EP	qCvM	BB	ES	BR

Table 23: Power, continuous data, D=2, with estimation

11.1.3 Continuous data, no estimation, D=5

uniform.uniform-diagonal-n	qCvM	BR	qKS	qAD	qK
uniform.uniform-diagonal-b	qCvM	BR	qAD	qKS	qK
normal-ind.normal-corA	qCvM	qKS	qAD	qK	BR
normal-ind.normal-corB	qCvM	qKS	qK	qAD	BR
t-ind.t-cor	qCvM	qKS	qK	qAD	BR
uniform.Frank	qCvM	qKS	BR	qK	qAD
uniform.Clayton	qCvM	BR	qAD	qKS	qK
uniform.Gumbel	qCvM	qKS	qK	BR	qAD
uniform.Joe	qCvM	qKS	qK	BR	qAD
mix.uniform.Clayton	qCvM	qKS	qAD	BR	qK
mix.Clayton.Gumbel	qKS	qCvM	qK	qAD	BR
mix.uniform.Frank	qCvM	qKS	qK	BR	qAD
mix.Clayton.Frank	qK	BR	qAD	qKS	qCvM
mix.Frank.Gumbel	qKS	qK	qCvM	BR	qAD
mix.Frank.Joe	qKS	qK	qCvM	BR	qAD
normal.normal-shift-one.marginal	qKS	qK	qCvM	BR	qAD
normal.shift-all.marginal	qKS	qCvM	qK	BR	qAD
normal.normal-stretch-one.marginal	qK	qAD	qKS	qCvM	BR
normal.stretch-all.marginal	qK	qKS	qAD	qCvM	BR
normal.stretch-shift-one.marginal	qKS	qK	qCvM	BR	qAD
normal.stretch-shift-all.marginal	qKS	qCvM	qK	BR	qAD
uniform.rotated-uniform.marginal	BR	qK	qKS	qCvM	qAD
uniform.beta-one.marginal	BR	qCvM	qKS	qK	qAD
uniform.beta-two.marginal	BR	qK	qKS	qCvM	qAD
uniform.all-normal.marginal	qKS	qK	qCvM	BR	qAD
beta22.all-normal.marginal	qK	qKS	qCvM	BR	qAD
uniform.all-beta.marginal	BR	qKS	qK	qCvM	qAD
uniform.all-exp.marginal	qK	qKS	qCvM	BR	qAD
uniform.all-gamma.marginal	qK	qKS	qAD	qCvM	BR
uniform.different.marginal	qK	qKS	qCvM	qAD	BR

Table 24: Power, continuous data, D=5, no estimation

11.1.4 Continuous data, hybrid methods, no estimation, nMC=1, Dim=2

uniform.uniform-diagonal-n	MMD	AZ	ES	EP	NN5	AD
uniform.uniform-diagonal-b	MMD	AZ	EP	ES	NN5	AD
normal-ind.normal-cor	ES	EP	AZ	AD	MMD	NN5
t-ind.t-cor	EP	AD	ES	AZ	CvM	NN5
uniform.Frank	MMD	AZ	ES	EP	AD	CvM
uniform.Clayton	MMD	AZ	EP	ES	NN5	AD
uniform.Gumbel	AZ	MMD	ES	EP	AD	CvM
uniform.Galambos	EP	MMD	AZ	ES	NN5	AD
uniform.HuslerReiss	AZ	MMD	ES	EP	NN5	AD
uniform.Joe	AZ	MMD	EP	ES	NN5	CvM
mix.Clayton.Gumbel	ES	EP	MMD	AZ	NN5	CF1
mix.uniform.Frank	EP	ES	NN5	AZ	MMD	AD
mix.Clayton.Frank	NN5	EP	ES	MMD	CF1	CF4
mix.Frank.Gumbel	ES	EP	NN5	MMD	CF1	CF4
mix.Frank.Joe	ES	EP	MMD	NN5	AZ	CF1
normal.normal-shift-one.marginal	BF	MMD	AZ	Ball	KS	CvM
normal.normal-shift-both.marginal	BF	AD	CvM	MMD	AZ	KS
normal.normal-stretch-one.marginal	BG	Ball	AZ	MMD	ES	K
normal.normal-stretch-both.marginal	BG	Ball	AZ	MMD	BF	K
normal.stretch-shift-one.marginal	BF	MMD	AZ	Ball	KS	CvM
normal.stretch-shift-two.marginal	CvM	BF	Ball	KS	AZ	AD
uniform.rotated-uniform.marginal	MMD	BF	AZ	NN5	Ball	K
uniform.beta-one.marginal	CvM	Ball	AD	BF	KS	AZ
uniform.beta-two.marginal	BG	Ball	K	AZ	MMD	KS
uni-exp-1.uni-exp-l.marginal	KS	CvM	AD	ES	BG	EP
exp-exp-1.exp-exp-l.marginal	BF	MMD	AZ	CvM	KS	Ball
beta-nor-1.beta-nor-mean.marginal	BG	Ball	AZ	MMD	K	ES
beta-nor-1.beta-nor-sd.marginal	BG	Ball	AZ	MMD	ES	BF
beta-beta-2.beta-beta-a.marginal	BG	K	Ball	AZ	ES	MMD
beta05.normal.marginal	ES	BG	K	EP	NN5	MMD

uniform.uniform-diagonal-n	CvM	Ball	BF	K	CF1	NN1
uniform.uniform-diagonal-b	CF1	BF	CvM	CF4	NN1	Ball
normal-ind.normal-cor	CvM	BF	KS	CF1	NN1	K
t-ind.t-cor	KS	MMD	K	BF	CF1	NN1
uniform.Frank	NN5	KS	BF	K	Ball	CF1
uniform.Clayton	CvM	BF	Ball	CF1	NN1	KS
uniform.Gumbel	NN5	BF	K	KS	CF1	Ball
uniform.Galambos	CvM	BF	KS	Ball	CF1	K
uniform.HuslerReiss	CvM	KS	BF	Ball	K	NN1
uniform.Joe	AD	KS	BF	CF1	Ball	CF4
mix.Clayton.Gumbel	NN1	AD	CF4	K	CvM	KS
mix.uniform.Frank	K	CF1	CvM	KS	CF4	NN1
mix.Clayton.Frank	NN1	CF2	AZ	NN0	K	AD
mix.Frank.Gumbel	NN1	AZ	NN0	CF2	K	KS
mix.Frank.Joe	AD	NN1	CF4	CvM	K	KS
normal.normal-shift-one.marginal	AD	K	ES	EP	NN5	CF1
normal.normal-shift-both.marginal	Ball	ES	EP	K	NN5	CF1
normal.normal-stretch-one.marginal	BF	AD	EP	NN5	KS	CvM
normal.normal-stretch-both.marginal	ES	AD	KS	NN5	EP	CvM
normal.stretch-shift-one.marginal	AD	K	ES	EP	BG	NN5
normal.stretch-shift-two.marginal	MMD	BG	K	ES	EP	NN5
uniform.rotated-uniform.marginal	NN1	CF1	ES	CF4	AD	EP
uniform.beta-one.marginal	MMD	BG	K	EP	ES	NN5
uniform.beta-two.marginal	AD	ES	EP	NN5	CvM	BF
uni-exp-1.uni-exp-l.marginal	BF	K	AZ	MMD	NN5	Ball
exp-exp-1.exp-exp-l.marginal	AD	ES	BG	EP	K	NN5
beta-nor-1.beta-nor-mean.marginal	BF	AD	EP	NN5	KS	CvM
beta-nor-1.beta-nor-sd.marginal	K	AD	EP	CvM	NN5	KS
beta-beta-2.beta-beta-a.marginal	NN5	AD	EP	BF	KS	CvM
beta05.normal.marginal	AZ	CF1	CF4	NN1	AD	KS

uniform.uniform-diagonal-n	KS	CF4	NN0	CF2	BG
uniform.uniform-diagonal-b	KS	NN0	K	CF2	BG
normal-ind.normal-cor	CF4	NN0	Ball	CF2	BG
t-ind.t-cor	CF4	NN0	Ball	CF2	BG
uniform.Frank	CF4	NN1	NN0	CF2	BG

uniform.Clayton	K	CF4	NN0	CF2	BG
uniform.Gumbel	CF4	NN1	NN0	CF2	BG
uniform.Galambos	CF4	NN0	NN1	CF2	BG
uniform.HuslerReiss	CF1	CF4	NN0	CF2	BG
uniform.Joe	K	NN1	CF2	NN0	BG
mix.Clayton.Gumbel	NN0	CF2	Ball	BF	BG
mix.uniform.Frank	NN0	CF2	Ball	BG	BF
mix.Clayton.Frank	CvM	KS	BF	Ball	BG
mix.Frank.Gumbel	AD	CvM	BG	Ball	BF
mix.Frank.Joe	CF2	NN0	BG	Ball	BF
normal.normal-shift-one.marginal	CF4	NN1	NN0	CF2	BG
normal.normal-shift-both.marginal	NN1	CF4	NN0	BG	CF2
normal.normal-stretch-one.marginal	NN1	CF1	CF4	CF2	NN0
normal.normal-stretch-both.marginal	CF1	CF4	NN1	CF2	NN0
normal.stretch-shift-one.marginal	CF1	CF4	NN1	NN0	CF2
normal.stretch-shift-two.marginal	CF1	CF4	NN1	CF2	NN0
uniform.rotated-uniform.marginal	NN0	KS	CF2	CvM	BG
uniform.beta-one.marginal	CF1	NN0	CF4	NN1	CF2
uniform.beta-two.marginal	CF4	NN1	CF1	NN0	CF2
uni-exp-1.uni-exp-l.marginal	NN1	CF1	CF4	NN0	CF2
exp-exp-1.exp-exp-l.marginal	CF1	NN1	NN0	CF4	CF2
beta-nor-1.beta-nor-mean.marginal	CF1	NN1	CF4	NN0	CF2
beta-nor-1.beta-nor-sd.marginal	CF1	CF4	NN0	NN1	CF2
beta-beta-2.beta-beta-a.marginal	CF1	NN1	CF4	NN0	CF2
beta05.normal.marginal	CF2	CvM	Ball	NN0	BF

Table 25: Power, continuous data, hybrid methods, nMC=1, D=2, no estimation

11.1.5 Continuous data, hybrid methods, no estimation, nMC=5, Dim=2

uniform.uniform-diagonal-n	MMD	AZ	EP	ES	AD	BF
uniform.uniform-diagonal-b	MMD	AZ	ES	EP	NN5	AD
normal-ind.normal-cor	EP	ES	AZ	AD	MMD	CvM
t-ind.t-cor	EP	AD	AZ	CvM	ES	MMD
uniform.Frank	MMD	AZ	ES	EP	AD	BF
uniform.Clayton	MMD	AZ	ES	EP	AD	BF
uniform.Gumbel	AZ	MMD	ES	EP	AD	BF
uniform.Galambos	AZ	MMD	ES	EP	AD	BF
uniform.HuslerReiss	AZ	MMD	EP	ES	AD	BF
uniform.Joe	AZ	MMD	ES	EP	BF	NN5
mix.Clayton.Gumbel	EP	ES	MMD	AZ	NN5	K
mix.uniform.Frank	MMD	AZ	ES	EP	AD	CvM
mix.Clayton.Frank	EP	ES	MMD	NN5	CF1	AZ
mix.Frank.Gumbel	ES	EP	NN5	MMD	AZ	CF1
mix.Frank.Joe	EP	ES	MMD	AZ	NN5	K
normal.normal-shift-one.marginal	BF	MMD	AZ	KS	CvM	AD
normal.normal-shift-both.marginal	BF	MMD	AD	CvM	AZ	KS
normal.normal-stretch-one.marginal	BG	ES	AZ	MMD	K	BF
normal.normal-stretch-both.marginal	BG	AZ	ES	MMD	BF	K
normal.stretch-shift-one.marginal	BF	MMD	AZ	KS	CvM	ES
normal.stretch-shift-two.marginal	CvM	BF	KS	MMD	AD	AZ
uniform.rotated-uniform.marginal	MMD	BF	AZ	NN5	K	CF1
uniform.beta-one.marginal	CvM	KS	AD	BF	MMD	AZ
uniform.beta-two.marginal	BG	K	MMD	AZ	AD	ES
uni-exp-1.uni-exp-l.marginal	KS	CvM	AD	K	EP	AZ
exp-exp-1.exp-exp-l.marginal	BF	MMD	AZ	KS	CvM	AD
beta-nor-1.beta-nor-mean.marginal	BG	AZ	MMD	K	ES	EP
beta-nor-1.beta-nor-sd.marginal	BG	AZ	MMD	K	BF	ES
beta-beta-2.beta-beta-a.marginal	BG	K	MMD	AZ	ES	EP
beta05.normal.marginal	ES	EP	K	BG	MMD	NN5
uniform.uniform-diagonal-n	NN5	CvM	KS	CF1	CF4	K
uniform.uniform-diagonal-b	BF	CvM	NN1	CF4	CF1	NN0

normal-ind.normal-cor	KS	BF	K	NN5	CF1	NN1
t-ind.t-cor	KS	NN5	K	BF	NN1	CF1
uniform.Frank	CvM	KS	NN5	K	CF1	CF4
uniform.Clayton	CvM	NN5	KS	CF1	CF4	K
uniform.Gumbel	CvM	NN5	KS	K	NN1	CF1
uniform.Galambos	CvM	NN5	KS	K	CF4	NN1
uniform.HuslerReiss	CvM	KS	NN5	K	CF4	NN1
uniform.Joe	CvM	AD	KS	K	NN1	NN0
mix.Clayton.Gumbel	AD	CvM	BF	KS	CF1	NN1
mix.uniform.Frank	KS	K	NN5	BF	BG	NN1
mix.Clayton.Frank	NN1	CF4	CF2	NN0	K	AD
mix.Frank.Gumbel	CF4	K	NN0	NN1	CF2	BG
mix.Frank.Joe	AD	CF1	CvM	KS	NN1	CF4
normal.normal-shift-one.marginal	K	ES	EP	NN5	CF1	CF4
normal.normal-shift-both.marginal	ES	EP	K	NN5	NN1	CF1
normal.normal-stretch-one.marginal	AD	EP	KS	NN5	CvM	CF1
normal.normal-stretch-both.marginal	AD	KS	EP	CvM	NN5	CF1
normal.stretch-shift-one.marginal	AD	K	EP	BG	NN5	CF1
normal.stretch-shift-two.marginal	BG	ES	K	EP	NN1	NN5
uniform.rotated-uniform.marginal	ES	NN1	CF4	NN0	CF2	AD
uniform.beta-one.marginal	K	BG	EP	ES	NN5	NN1
uniform.beta-two.marginal	EP	KS	CvM	BF	NN5	CF4
uni-exp-1.uni-exp-l.marginal	BF	MMD	BG	ES	NN5	NN1
exp-exp-1.exp-exp-l.marginal	EP	K	BG	ES	NN5	NN0
beta-nor-1.beta-nor-mean.marginal	BF	AD	KS	NN5	CvM	NN0
beta-nor-1.beta-nor-sd.marginal	AD	EP	CvM	KS	NN5	CF4
beta-beta-2.beta-beta-a.marginal	AD	KS	CvM	BF	NN5	NN1
beta05.normal.marginal	AZ	CF1	AD	KS	CF4	NN1
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	NN0	NN1	CF2	BG		
uniform.uniform-diagonal-b	KS	K	CF2	BG		
normal-ind.normal-cor	NN0	CF4	CF2	BG		
t-ind.t-cor	NN0	CF4	CF2	BG		
uniform.Frank	NN1	NN0	CF2	BG		
uniform.Clayton	NN0	NN1	CF2	BG		
uniform.Gumbel	NN0	CF4	CF2	BG		
uniform.Galambos	CF1	NN0	CF2	BG		
uniform.HuslerReiss	CF1	NN0	CF2	BG		
uniform.Joe	CF4	CF1	CF2	BG		
mix.Clayton.Gumbel	NN0	CF4	BG	CF2		
mix.uniform.Frank	CF4	CF1	NN0	CF2		
mix.Clayton.Frank	CvM	KS	BG	BF		
mix.Frank.Gumbel	KS	CvM	AD	BF		
mix.Frank.Joe	NN0	BF	CF2	BG		
normal.normal-shift-one.marginal	NN0	NN1	BG	CF2		
normal.normal-shift-both.marginal	CF4	NN0	CF2	BG		
normal.normal-stretch-one.marginal	CF4	NN1	NN0	CF2		
normal.normal-stretch-both.marginal	CF4	NN1	NN0	CF2		
normal.stretch-shift-one.marginal	NN1	NN0	CF4	CF2		
normal.stretch-shift-two.marginal	CF1	CF4	NN0	CF2		
uniform.rotated-uniform.marginal	EP	KS	CvM	BG		
uniform.beta-one.marginal	NN0	CF4	CF1	CF2		
uniform.beta-two.marginal	NN1	NN0	CF1	CF2		
uni-exp-1.uni-exp-l.marginal	NN0	CF4	CF1	CF2		
exp-exp-1.exp-exp-l.marginal	CF1	CF4	NN1	CF2		
beta-nor-1.beta-nor-mean.marginal	CF1	NN1	CF4	CF2		
beta-nor-1.beta-nor-sd.marginal	CF1	NN0	NN1	CF2		
beta-beta-2.beta-beta-a.marginal	CF1	CF4	NN0	CF2		
beta05.normal.marginal	CvM	CF2	NN0	BF		

Table 26: Power, continuous data, hybrid methods, nMC=5, D=2, no estimation

11.1.6 Continuous data, hybrid methods, no estimation, nMC=1, Dim=5

uniform.uniform-diagonal-n	MMD	AD	CvM	NN5	AZ	CF1
uniform.uniform-diagonal-b	MMD	AD	CvM	NN5	AZ	CF1
normal-ind.normal-corA	AD	CvM	NN5	KS	MMD	AZ
normal-ind.normal-corB	AD	CvM	NN5	KS	MMD	AZ
t-ind.t-cor	AD	CvM	KS	NN5	K	AZ
uniform.Frank	MMD	AD	CvM	NN5	AZ	KS
uniform.Clayton	MMD	AD	AZ	NN5	CvM	Ball
uniform.Gumbel	AD	CvM	MMD	NN5	KS	AZ
uniform.Joe	MMD	CvM	KS	AD	NN5	AZ
mix.uniform.Clayton	MMD	AZ	CvM	AD	NN5	KS
mix.Clayton.Gumbel	MMD	AZ	AD	CvM	NN5	KS
mix.uniform.Frank	MMD	CvM	AZ	AD	KS	NN5
mix.Clayton.Frank	MMD	NN5	NN1	CF1	CF4	NN0
mix.Frank.Gumbel	MMD	NN5	CF4	CF2	CF1	AD
mix.Frank.Joe	MMD	NN5	AD	AZ	CvM	CF4
normal.normal-shift-one.marginal	BF	AZ	MMD	Ball	KS	CvM
normal.shift-all.marginal	AZ	BF	MMD	CvM	Ball	AD
normal.normal-stretch-one.marginal	BG	Ball	MMD	NN5	K	AZ
normal.stretch-all.marginal	BG	Ball	AZ	MMD	BF	CF2
normal.stretch-shift-one.marginal	BF	AZ	MMD	Ball	KS	CvM
normal.stretch-shift-all.marginal	Ball	AZ	BF	MMD	CvM	KS
uniform.rotated-uniform.marginal	BF	MMD	AZ	Ball	NN5	CF1
uniform.beta-one.marginal	Ball	BF	CvM	AZ	MMD	KS
uniform.beta-two.marginal	BG	Ball	MMD	AZ	CF4	CF2
uniform.all-normal.marginal	AZ	BG	MMD	Ball	BF	CF2
beta22.all-normal.marginal	BG	Ball	AZ	MMD	BF	CF2
uniform.all-beta.marginal	BG	Ball	MMD	AZ	K	CF4
uniform.all-exp.marginal	BG	Ball	AZ	MMD	BF	K
uniform.all-gamma.marginal	BG	Ball	MMD	AZ	BF	CF2
uniform.different.marginal	BG	Ball	MMD	AZ	K	CF2

uniform.uniform-diagonal-n	CF4	KS	NN0	NN1	CF2	K
uniform.uniform-diagonal-b	CF4	KS	NN0	NN1	CF2	BF
normal-ind.normal-corA	NN0	NN1	CF1	CF4	BF	K
normal-ind.normal-corB	NN1	CF1	NN0	CF4	BF	K
t-ind.t-cor	NN1	CF1	MMD	CF4	NN0	BF
uniform.Frank	CF1	NN1	CF4	BF	NN0	CF2
uniform.Clayton	BF	CF4	CF2	CF1	NN0	KS
uniform.Gumbel	CF1	NN0	NN1	CF4	Ball	CF2
uniform.Joe	NN0	NN1	CF4	CF1	K	Ball
mix.uniform.Clayton	Ball	K	CF1	CF4	NN0	CF2
mix.Clayton.Gumbel	K	CF1	CF4	CF2	Ball	BF
mix.uniform.Frank	K	Ball	CF1	BF	NN0	CF4
mix.Clayton.Frank	CF2	K	AZ	AD	KS	BF
mix.Frank.Gumbel	CvM	NN1	KS	K	AZ	BG
mix.Frank.Joe	CF2	CF1	K	KS	NN1	Ball
normal.normal-shift-one.marginal	K	AD	NN5	CF1	NN1	CF4
normal.shift-all.marginal	KS	K	NN5	NN1	CF1	CF4
normal.normal-stretch-one.marginal	CF4	CF2	CF1	KS	CvM	AD
normal.stretch-all.marginal	CF4	K	KS	NN5	CF1	NN1
normal.stretch-shift-one.marginal	K	NN5	AD	BG	NN1	CF1
normal.stretch-shift-all.marginal	BG	AD	K	NN5	CF4	CF2
uniform.rotated-uniform.marginal	K	NN1	CF4	NN0	KS	AD
uniform.beta-one.marginal	AD	BG	K	NN5	CF4	CF1
uniform.beta-two.marginal	K	KS	NN0	NN5	BF	CvM
uniform.all-normal.marginal	CF4	K	NN0	NN5	KS	CF1
beta22.all-normal.marginal	CF4	NN5	K	KS	NN0	CF1
uniform.all-beta.marginal	CF2	KS	BF	NN5	NN0	CvM
uniform.all-exp.marginal	KS	CF2	CF4	NN5	NN0	CF1
uniform.all-gamma.marginal	CF4	K	KS	NN5	CF1	CvM
uniform.different.marginal	CF4	KS	NN5	BF	CvM	CF1

uniform.uniform-diagonal-n	BF	Ball	BG
uniform.uniform-diagonal-b	K	Ball	BG
normal-ind.normal-corA	CF2	Ball	BG
normal-ind.normal-corB	CF2	Ball	BG
t-ind.t-cor	CF2	Ball	BG

uniform.Frank	Ball	K	BG
uniform.Clayton	NN1	K	BG
uniform.Gumbel	BF	K	BG
uniform.Joe	BF	CF2	BG
mix.uniform.Clayton	NN1	BF	BG
mix.Clayton.Gumbel	NN1	BG	NN0
mix.uniform.Frank	NN1	BG	CF2
mix.Clayton.Frank	CvM	Ball	BG
mix.Frank.Gumbel	Ball	BF	NN0
mix.Frank.Joe	BG	NN0	BF
normal.normal-shift-one.marginal	NN0	CF2	BG
normal.shift-all.marginal	CF2	NN0	BG
normal.normal-stretch-one.marginal	NN1	BF	NN0
normal.stretch-all.marginal	CvM	AD	NN0
normal.stretch-shift-one.marginal	CF4	CF2	NN0
normal.stretch-shift-all.marginal	CF1	NN1	NN0
uniform.rotated-uniform.marginal	CF2	CvM	BG
uniform.beta-one.marginal	NN0	CF2	NN1
uniform.beta-two.marginal	CF1	NN1	AD
uniform.all-normal.marginal	NN1	CvM	AD
beta22.all-normal.marginal	NN1	CvM	AD
uniform.all-beta.marginal	NN1	CF1	AD
uniform.all-exp.marginal	CvM	NN1	AD
uniform.all-gamma.marginal	NN1	AD	NN0
uniform.different.marginal	NN1	AD	NN0

Table 27: Power, continuous data, hybrid methods, nMC=1, D=5, no estimation

11.1.7 Continuous data, hybrid methods, no estimation, nMC=5, Dim=5

uniform.uniform-diagonal-n	MMD	AD	NN5	CvM	CF4	NN1
uniform.uniform-diagonal-b	MMD	AD	NN5	CvM	AZ	CF4
normal-ind.normal-corA	AD	CvM	KS	AZ	NN5	MMD
normal-ind.normal-corB	AD	CvM	KS	MMD	NN5	AZ
t-ind.t-cor	AD	CvM	KS	NN5	AZ	MMD
uniform.Frank	AD	MMD	CvM	AZ	NN5	KS
uniform.Clayton	CvM	AD	NN5	AZ	BF	MMD
uniform.Gumbel	MMD	AD	CvM	AZ	NN5	KS
uniform.Joe	MMD	CvM	NN5	AD	AZ	KS
mix.uniform.Clayton	MMD	AZ	CvM	AD	KS	BF
mix.Clayton.Gumbel	MMD	AZ	CvM	AD	KS	K
mix.uniform.Frank	MMD	AZ	CvM	AD	KS	K
mix.Clayton.Frank	MMD	NN5	NN1	NN0	CF4	CF1
mix.Frank.Gumbel	MMD	CF1	NN5	AZ	CF2	CF4
mix.Frank.Joe	MMD	AZ	AD	CvM	NN5	KS
normal.normal-shift-one.marginal	BF	AZ	MMD	KS	CvM	K
normal.shift-all.marginal	KS	CvM	AZ	BF	MMD	AD
normal.normal-stretch-one.marginal	BG	MMD	AZ	CF1	BF	NN5
normal.stretch-all.marginal	AZ	BG	MMD	BF	CF1	K
normal.stretch-shift-one.marginal	BF	AZ	MMD	KS	CvM	K
normal.stretch-shift-all.marginal	KS	AZ	BF	MMD	CvM	BG
uniform.rotated-uniform.marginal	BF	MMD	AZ	NN5	K	CF1
uniform.beta-one.marginal	KS	AZ	BF	MMD	CvM	AD
uniform.beta-two.marginal	BG	MMD	AZ	K	KS	CF4
uniform.all-normal.marginal	AZ	BF	BG	MMD	K	KS
beta22.all-normal.marginal	AZ	BG	MMD	BF	K	KS
uniform.all-beta.marginal	BG	MMD	K	KS	AZ	CF4
uniform.all-exp.marginal	AZ	BG	MMD	BF	K	KS
uniform.all-gamma.marginal	AZ	BG	MMD	BF	K	CF1
uniform.different.marginal	BG	AZ	MMD	K	KS	CF1
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	AZ	NN0	CF2	KS	CF1	BF
uniform.uniform-diagonal-b	NN1	NN0	CF2	KS	CF1	BF

normal-ind.normal-corA	NN1	BF	K	CF4	NN0	CF1
normal-ind.normal-corB	CF4	NN0	K	NN1	BF	CF1
t-ind.t-cor	NN1	K	CF4	NN0	BF	CF2
uniform.Frank	BF	CF4	NN0	NN1	CF1	CF2
uniform.Clayton	CF4	NN0	CF2	KS	NN1	CF1
uniform.Gumbel	BF	NN1	NN0	CF4	K	CF2
uniform.Joe	NN1	NN0	CF4	BF	K	CF2
mix.uniform.Clayton	K	NN5	BG	NN0	CF4	NN1
mix.Clayton.Gumbel	NN5	CF1	BF	CF4	CF2	BG
mix.uniform.Frank	NN5	BF	BG	CF4	NN0	NN1
mix.Clayton.Frank	K	CF2	AZ	KS	BG	AD
mix.Frank.Gumbel	AD	CvM	K	KS	NN1	BG
mix.Frank.Joe	K	CF1	CF4	CF2	NN1	NN0
normal.normal-shift-one.marginal	NN5	AD	NN1	NN0	CF4	CF1
normal.shift-all.marginal	K	NN5	CF1	NN1	CF4	NN0
normal.normal-stretch-one.marginal	K	CF4	CF2	NN1	AD	KS
normal.stretch-all.marginal	CF2	CF4	KS	NN5	CvM	AD
normal.stretch-shift-one.marginal	BG	NN5	CF1	CF4	AD	NN1
normal.stretch-shift-all.marginal	K	AD	CF1	NN5	CF2	CF4
uniform.rotated-uniform.marginal	AD	CF4	NN1	CF2	KS	NN0
uniform.beta-one.marginal	K	BG	NN5	CF4	NN1	NN0
uniform.beta-two.marginal	CF2	CvM	BF	NN5	NN0	NN1
uniform.all-normal.marginal	CF2	CF4	NN5	CvM	NN0	NN1
beta22.all-normal.marginal	CF2	CF4	NN5	CvM	NN0	NN1
uniform.all-beta.marginal	CF2	BF	NN5	CvM	NN0	NN1
uniform.all-exp.marginal	NN5	CvM	CF2	CF4	NN0	NN1
uniform.all-gamma.marginal	KS	CF2	CF4	NN5	AD	CvM
uniform.different.marginal	BF	CF4	CF2	CvM	AD	NN5

uniform.uniform-diagonal-n	K	BG
uniform.uniform-diagonal-b	K	BG
normal-ind.normal-corA	CF2	BG
normal-ind.normal-corB	CF2	BG
t-ind.t-cor	CF1	BG
uniform.Frank	K	BG
uniform.Clayton	K	BG
uniform.Gumbel	CF1	BG
uniform.Joe	CF1	BG
mix.uniform.Clayton	CF2	CF1
mix.Clayton.Gumbel	NN1	NN0
mix.uniform.Frank	CF2	CF1
mix.Clayton.Frank	CvM	BF
mix.Frank.Gumbel	NN0	BF
mix.Frank.Joe	BG	BF
normal.normal-shift-one.marginal	CF2	BG
normal.shift-all.marginal	CF2	BG
normal.normal-stretch-one.marginal	NN0	CvM
normal.stretch-all.marginal	NN1	NN0
normal.stretch-shift-one.marginal	NN0	CF2
normal.stretch-shift-all.marginal	NN1	NN0
uniform.rotated-uniform.marginal	BG	CvM
uniform.beta-one.marginal	CF2	CF1
uniform.beta-two.marginal	AD	CF1
uniform.all-normal.marginal	AD	CF1
beta22.all-normal.marginal	AD	CF1
uniform.all-beta.marginal	AD	CF1
uniform.all-exp.marginal	AD	CF1
uniform.all-gamma.marginal	NN1	NN0
uniform.different.marginal	NN1	NN0

Table 28: Power, continuous data, hybrid methods, nMC=5, D=5, no estimation

11.1.8 Continuous data, gof+hybrid methods, no estimation, Dim=2

uniform.uniform-diagonal-n	EPG	ESG	MMDT5	EPT5	EST5	MMDT1
uniform.uniform-diagonal-b	EPG	ESG	MMDT5	EST5	EPT5	qADG
normal-ind.normal-cor	KSDG	ESG	EPG	EPT5	EST5	qADG
t-ind.t-cor	EPG	KSDG	EPT5	ESG	qADG	qCvMG
uniform.Frank	ESG	EPG	MMDT5	EST5	EPT5	qADG
uniform.Clayton	ESG	EPG	qADG	MMDT5	EST5	EPT5
uniform.Gumbel	ESG	EPG	MMDT5	EST5	EPT5	qADG
uniform.Galambos	ESG	EPG	MMDT5	EST5	EPT5	qCvMG
uniform.HuslerReiss	EPG	ESG	MMDT5	EPT5	EST5	qADG
uniform.Joe	EPG	ESG	MMDT5	EST5	EPT5	FFG
mix.Clayton.Gumbel	ESG	EPG	RKG	EPT5	EST5	MMDT5
mix.uniform.Frank	ESG	EPG	MMDT5	RKG	FFG	EST5
mix.Clayton.Frank	RKG	ESG	EPG	FFG	EPT5	EST5
mix.Frank.Gumbel	RKG	EPG	ESG	EST5	EPT5	FFG
mix.Frank.Joe	RKG	ESG	EPG	EPT5	EST5	FFG
normal.normal-shift-one.marginal	FFG	BFT5	KSDG	MMDT5	qKSG	KST5
normal.normal-shift-both.marginal	qCvMG	BFT5	MMDT5	ADT5	KSDG	CvMT5
normal.normal-stretch-one.marginal	KSDG	BGT5	EST5	qADG	RKG	qKG
normal.normal-stretch-both.marginal	BGT5	BGT1	KSDG	RKG	EST5	BallT1
normal.stretch-shift-one.marginal	KSDG	BFT5	FFG	MMDT5	qKSG	KST5
normal.stretch-shift-two.marginal	qKSG	qCvMG	CvMT5	KSDG	BFT5	KST5
uniform.rotated-uniform.marginal	FFG	MMDT5	BFT5	NN5T5	BRG	MMDT1
uniform.beta-one.marginal	qCvMG	qKSG	FFG	CvMT5	qADG	KST5
uniform.beta-two.marginal	KSDG	BGT5	BRG	RKG	BGT1	qKG
uni-exp-1.uni-exp-l.marginal	KSDG	FFG	qKSG	qCvMG	KST5	CvMT5
exp-exp-1.exp-exp-l.marginal	FFG	qKSG	BFT5	qCvMG	MMDT5	KST5
beta-nor-1.beta-nor-mean.marginal	BGT5	qKG	BGT1	MMDT5	ESG	KT5
beta-nor-1.beta-nor-sd.marginal	BGT5	MMDT5	BGT1	qKG	ESG	BallT1
beta-beta-2.beta-beta-a.marginal	BGT5	qKG	KT5	RKG	BGT1	MMDT5
beta05.normal.marginal	EST5	RKG	ESG	BRG	qKG	EPG
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	FFG	qADG	qCvMG	KSDG	ADT5	EST1
uniform.uniform-diagonal-b	MMDT1	NN5T5	ADT5	qCvMG	BRG	FFG
normal-ind.normal-cor	qCvMG	FFG	ADT5	MMDT5	EST1	EPT1
t-ind.t-cor	ADT5	FFG	CvMT5	qKSG	EST5	EPT1
uniform.Frank	FFG	qCvMG	BRG	ADT5	MMDT1	EST1
uniform.Clayton	ADT5	qCvMG	FFG	MMDT1	EPT1	EST1
uniform.Gumbel	FFG	qCvMG	MMDT1	EST1	ADT5	BRG
uniform.Galambos	qADG	EPT1	FFG	MMDT1	BRG	ADT5
uniform.HuslerReiss	FFG	qCvMG	ADT5	MMDT1	BFT5	EST1
uniform.Joe	MMDT1	qCvMG	EPT1	EST1	BFT5	qADG
mix.Clayton.Gumbel	FFG	BRG	EST1	EPT1	MMDT1	NN5T5
mix.uniform.Frank	EPT5	KSDG	EPT1	EST1	qCvMG	qADG
mix.Clayton.Frank	MMDT5	NN5T5	NN5T1	EPT1	EST1	MMDT1
mix.Frank.Gumbel	EST1	NN5T5	MMDT5	EPT1	NN5T1	BRG
mix.Frank.Joe	MMDT5	EST1	EPT1	NN5T5	MMDT1	qKG
normal.normal-shift-one.marginal	qCvMG	CvMT5	ESG	BFT1	ADT5	qKG
normal.normal-shift-both.marginal	qKSG	FFG	KST5	qADG	BFT1	ADT1
normal.normal-stretch-one.marginal	MMDT5	ESG	BGT1	KT5	BFT5	ADT5
normal.normal-stretch-both.marginal	MMDT5	ESG	qADG	qKG	BFT5	FFG
normal.stretch-shift-one.marginal	ESG	qKG	CvMT5	EST5	qCvMG	BFT1
normal.stretch-shift-two.marginal	FFG	MMDT5	ADT5	ESG	qADG	qKG
uniform.rotated-uniform.marginal	qKG	KT5	CF1T5	CF3T5	BFT1	EST5
uniform.beta-one.marginal	BRG	ADT5	BFT5	MMDT5	qKG	CvMT1
uniform.beta-two.marginal	KT5	MMDT5	FFG	BallT1	EPG	ESG
uni-exp-1.uni-exp-l.marginal	qADG	ADT5	BRG	ESG	EPG	KT5
exp-exp-1.exp-exp-l.marginal	CvMT5	ADT5	ESG	BFT1	EPG	qADG
beta-nor-1.beta-nor-mean.marginal	RKG	BallT1	EPG	EST5	EPT5	MMDT1
beta-nor-1.beta-nor-sd.marginal	KT5	RKG	BFT5	EST5	EPG	ADT5
beta-beta-2.beta-beta-a.marginal	BRG	ESG	EPG	EST5	EPT5	FFG
beta05.normal.marginal	EPT5	KT5	BGT5	FFG	qADG	EST1
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	EPT1	BRG	BFT5	RKG	NN5T5	qKSG
uniform.uniform-diagonal-b	EPT1	EST1	BFT5	NN5T1	KSDG	CvMT5
normal-ind.normal-cor	CvMT5	qKSG	ADT1	MMDT1	NN5T1	KST5
t-ind.t-cor	MMDT5	ADT1	EST1	KST5	CvMT1	NN5T1
uniform.Frank	BFT5	KSDG	EPT1	CvMT5	qKSG	ADT1

uniform.Clayton	BRG	KSDG	BFT5	CvMT5	qKSG	NN5T5
uniform.Gumbel	KSDG	EPT1	BFT5	qKSG	CvMT5	ADT1
uniform.Galambos	EST1	BFT5	KSDG	CvMT5	NN5T5	qKSG
uniform.HuslerReiss	EPT1	CvMT5	KSDG	BRG	qKSG	KST5
uniform.Joe	KSDG	BRG	NN5T5	qKSG	CvMT5	ADT5
mix.Clayton.Gumbel	NN5T1	qKG	KT5	ADT5	CvMT5	BFT5
mix.uniform.Frank	ADT5	BRG	CvMT5	qKSG	KST5	KT5
mix.Clayton.Frank	CF1T5	CF3T5	CF1T1	CF4T1	NN1T5	NN1T1
mix.Frank.Gumbel	CF3T5	CF1T5	qKG	MMDT1	CF4T5	CF1T1
mix.Frank.Joe	BRG	NN5T1	KT5	qKSG	ADT5	CF3T5
normal.normal-shift-one.marginal	MMDT1	EPG	BallT1	KT5	qADG	EST5
normal.normal-shift-both.marginal	CvMT1	ESG	MMDT1	EPG	KST1	BallT1
normal.normal-stretch-one.marginal	BallT1	FFG	EPG	qKSG	EPT5	MMDT1
normal.normal-stretch-both.marginal	KT5	MMDT1	ADT5	EPG	qKSG	qCvMG
normal.stretch-shift-one.marginal	ADT5	MMDT1	EPG	BallT1	KT5	KST1
normal.stretch-shift-two.marginal	CvMT1	BGT5	EST5	BFT1	BallT1	KST1
uniform.rotated-uniform.marginal	NN1T5	CF4T5	NN5T1	NN0T5	BallT1	CF2T5
uniform.beta-one.marginal	KSDG	BallT1	ADT1	BFT1	KST1	MMDT1
uniform.beta-two.marginal	KT1	ADT5	EST5	EPT5	qKSG	KST5
uni-exp-1.uni-exp-l.marginal	qKG	KST1	CvMT1	EPT5	ADT1	EST1
exp-exp-1.exp-exp-l.marginal	MMDT1	CvMT1	KST1	qKG	BallT1	ADT1
beta-nor-1.beta-nor-mean.marginal	KT1	BFT5	EST1	FFG	qKSG	ADT5
beta-nor-1.beta-nor-sd.marginal	MMDT1	qCvMG	EPT5	EST1	CvMT5	FFG
beta-beta-2.beta-beta-a.marginal	KT1	BallT1	ADT5	qKSG	EST1	MMDT1
beta05.normal.marginal	MMDT5	NN5T5	BGT1	KT1	qKSG	EPT1
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	CvMT5	NN5T1	KST5	ADT1	CF3T5	qKG
uniform.uniform-diagonal-b	CF3T5	NN1T5	CF4T5	qKSG	ADT1	CF1T5
normal-ind.normal-cor	CvMT1	BFT5	KT5	NN5T5	BFT1	qKG
t-ind.t-cor	NN5T5	KST1	KT5	BFT5	MMDT1	qKG
uniform.Frank	KST5	CvMT1	qKG	NN5T5	NN5T1	KST1
uniform.Clayton	NN5T1	ADT1	CF3T5	CvMT1	qKG	BFT1
uniform.Gumbel	NN5T5	CvMT1	KST5	NN5T1	KT5	BFT1
uniform.Galambos	NN5T1	ADT1	KST5	CvMT1	BFT1	KST1
uniform.HuslerReiss	NN5T5	NN5T1	ADT1	CvMT1	KST1	KT5
uniform.Joe	NN5T1	KST5	CvMT1	ADT1	qKG	KT5
mix.Clayton.Gumbel	qKSG	KST5	CF1T5	CF1T1	CF3T5	NN1T5
mix.uniform.Frank	NN5T1	qKG	MMDT1	NN5T5	BFT5	ADT1
mix.Clayton.Frank	CF4T5	BBG	CF2T1	CF2T5	NN0T5	NN0T1
mix.Frank.Gumbel	KT5	NN0T5	CF4T1	qKSG	NN1T5	CF2T5
mix.Frank.Joe	CF1T5	CvMT5	CF1T1	KST5	ADT1	NN1T1
normal.normal-shift-one.marginal	KST1	CvMT1	EPT5	ADT1	KT1	EST1
normal.normal-shift-both.marginal	EST5	qKG	EPT5	KT5	EST1	BRG
normal.normal-stretch-one.marginal	EST1	KT1	qCvMG	KST5	NN5T5	CvMT5
normal.normal-stretch-both.marginal	BFT1	KT1	EST1	KST5	EPT5	CvMT5
normal.stretch-shift-one.marginal	EPT5	CvMT1	qADG	ADT1	BGT5	KT1
normal.stretch-shift-two.marginal	ADT1	MMDT1	EPG	KT5	RKG	EPT5
uniform.rotated-uniform.marginal	ADT5	EPT5	KT1	qADG	NN1T1	qKSG
uniform.beta-one.marginal	EPG	ESG	KT5	BGT5	EPT5	EST5
uniform.beta-two.marginal	CvMT5	MMDT1	qCvMG	KST1	BFT5	ADT1
uni-exp-1.uni-exp-l.marginal	BFT5	MMDT5	BGT5	BGT1	EPT1	BFT1
exp-exp-1.exp-exp-l.marginal	EPT5	KT5	BGT5	EST1	EST5	BGT1
beta-nor-1.beta-nor-mean.marginal	BFT1	ADT1	EPT1	KST5	NN5T1	qCvMG
beta-nor-1.beta-nor-sd.marginal	BFT1	qKSG	KT1	KST5	ADT1	EPT1
beta-beta-2.beta-beta-a.marginal	KST5	qCvMG	CvMT5	BFT5	NN5T1	ADT1
beta05.normal.marginal	NN5T1	CF1T5	ADT5	KST5	MMDT1	CF3T5
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	CvMT1	CF1T5	CF4T5	BallT1	BFT1	KT1
uniform.uniform-diagonal-b	NN0T5	CF1T1	KST5	BFT1	qKG	CvMT1
normal-ind.normal-cor	KST1	CF1T1	CF3T5	BBG	NN1T1	KT1
t-ind.t-cor	KT1	NN1T5	BFT1	CF1T1	NN1T1	CF1T5
uniform.Frank	BFT1	KT1	KT5	BallT1	CF3T5	CF1T1
uniform.Clayton	BallT1	KST5	CF1T1	NN1T1	CF1T5	CF4T5
uniform.Gumbel	KT1	CF3T5	KST1	CF1T1	NN1T5	qKG
uniform.Galambos	CF3T5	KT5	qKG	BallT1	CF1T1	CF4T5
uniform.HuslerReiss	BFT1	CF3T5	qKG	BallT1	KT1	NN1T1
uniform.Joe	CF3T5	KST1	BFT1	CF1T1	NN1T5	BallT1

mix.Clayton.Gumbel	qCvMG	NN1T1	qADG	ADT1	CF4T1	NN0T5
mix.uniform.Frank	KT1	CF1T1	CvMT1	BGT5	CF3T5	KST1
mix.Clayton.Frank	qKG	KT5	ADT5	CvMT5	KT1	ADT1
mix.Frank.Gumbel	BGT5	NN1T1	KST5	qCvMG	CvMT5	NN0T1
mix.Frank.Joe	NN1T5	CF4T5	CF4T1	NN0T5	qCvMG	BFT5
normal.normal-shift-one.marginal	EPT1	NN5T1	NN5T5	CF3T5	CF1T1	BBG
normal.normal-shift-both.marginal	EPT1	KT1	NN5T5	NN5T1	NN1T5	CF3T5
normal.normal-stretch-one.marginal	BFT1	ADT1	EPT1	CF1T5	CF3T5	NN5T1
normal.normal-stretch-both.marginal	NN5T5	ADT1	KST1	NN5T1	EPT1	CF1T5
normal.stretch-shift-one.marginal	EST1	RKG	EPT1	BGT1	BRG	NN5T5
normal.stretch-shift-two.marginal	BGT1	BRG	KT1	EST1	EPT1	NN5T1
uniform.rotated-uniform.marginal	CF1T1	KST5	EST1	CF4T1	qCvMG	ADT1
uniform.beta-one.marginal	BGT1	KT1	EPT1	EST1	RKG	NN5T5
uniform.beta-two.marginal	EST1	EPT1	NN5T5	NN5T1	CvMT1	BFT1
uni-exp-1.uni-exp-l.marginal	KT1	EST5	MMDT1	NN5T5	NN5T1	BallT1
exp-exp-1.exp-exp-l.marginal	EPT1	KSDG	KT1	NN5T1	NN5T5	RKG
beta-nor-1.beta-nor-mean.marginal	KST1	NN5T5	CvMT5	CvMT1	CF1T1	CF3T5
beta-nor-1.beta-nor-sd.marginal	CvMT1	NN5T1	BRG	KST1	NN5T5	CF1T1
beta-beta-2.beta-beta-a.marginal	EPT1	NN5T5	BFT1	KST1	CF3T5	CvMT1
beta05.normal.marginal	CF4T5	NN1T5	CvMT5	CF2T5	NN0T5	qCvMG
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	CF1T1	KT5	NN0T5	NN1T5	NN1T1	KST1
uniform.uniform-diagonal-b	KT5	CF2T5	RKG	CF4T1	NN1T1	BallT1
normal-ind.normal-cor	CF4T1	CF1T5	NN1T5	NN0T5	NN0T1	CF4T5
t-ind.t-cor	CF3T5	NN0T5	BBG	CF4T1	CF4T5	NN0T1
uniform.Frank	CF1T5	CF4T5	NN1T5	CF4T1	NN0T5	NN1T1
uniform.Clayton	KT5	NN0T5	KST1	KT1	NN1T5	CF4T1
uniform.Gumbel	CF1T5	NN0T5	BallT1	CF4T5	CF4T1	NN1T1
uniform.Galambos	NN1T5	CF1T5	NN0T5	KT1	CF4T1	NN0T1
uniform.HuslerReiss	CF1T1	CF4T5	NN1T5	CF1T5	NN0T5	CF4T1
uniform.Joe	NN0T5	CF4T5	CF4T1	KT1	CF1T5	NN1T1
mix.Clayton.Gumbel	CF4T5	KT1	CvMT1	BGT5	KST1	NN0T1
mix.uniform.Frank	CF4T1	NN1T5	NN1T1	CF4T5	NN0T1	CF2T1
mix.Clayton.Frank	CvMT1	KST1	qKSG	KST5	BFT1	BGT5
mix.Frank.Gumbel	ADT5	CF2T1	KT1	BBG	KST1	ADT1
mix.Frank.Joe	CvMT1	KT1	KST1	BBG	CF2T5	CF2T1
normal.normal-shift-one.marginal	CF1T5	CF4T5	NN0T5	CF4T1	NN1T1	NN1T5
normal.normal-shift-both.marginal	CF1T1	BBG	CF1T5	NN1T1	CF4T5	NN0T5
normal.normal-stretch-one.marginal	KST1	CvMT1	BRG	CF4T5	NN1T5	BBG
normal.normal-stretch-both.marginal	CF3T5	CvMT1	CF1T1	CF4T5	CF4T1	BRG
normal.stretch-shift-one.marginal	NN5T1	CF1T5	CF3T5	NN1T5	CF1T1	NN0T5
normal.stretch-shift-two.marginal	NN1T5	NN5T5	CF1T5	CF3T5	CF4T5	CF1T1
uniform.rotated-uniform.marginal	EPT1	EPG	NN0T1	KST1	CvMT5	ESG
uniform.beta-one.marginal	NN5T1	CF3T5	CF1T1	NN0T1	NN1T5	NN0T5
uniform.beta-two.marginal	CF3T5	CF4T1	NN1T1	CF1T1	CF4T5	NN0T1
uni-exp-1.uni-exp-l.marginal	RKG	CF3T5	NN1T5	NN1T1	NN0T5	CF1T1
exp-exp-1.exp-exp-l.marginal	CF1T1	CF3T5	NN1T1	NN0T5	CF1T5	NN0T1
beta-nor-1.beta-nor-mean.marginal	NN1T1	CF4T1	NN0T5	CF1T5	NN1T5	BRG
beta-nor-1.beta-nor-sd.marginal	CF3T5	CF4T1	CF4T5	CF1T5	NN0T5	NN0T1
beta-beta-2.beta-beta-a.marginal	CF1T1	NN1T1	NN1T5	CF4T1	CF1T5	CF4T5
beta05.normal.marginal	CF1T1	CF4T1	NN1T1	ADT1	KST1	CF2T1
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	CF2T5	CF4T1	NN0T1	CF2T1	BGT5	BBG
uniform.uniform-diagonal-b	KST1	NN0T1	KT1	BBG	CF2T1	BGT1
normal-ind.normal-cor	BallT1	CF2T1	CF2T5	BGT5	RKG	BGT1
t-ind.t-cor	BRG	BallT1	CF2T5	CF2T1	RKG	BGT5
uniform.Frank	CF2T5	RKG	NN0T1	CF2T1	BGT5	BGT1
uniform.Clayton	NN0T1	CF2T5	CF2T1	RKG	BBG	BGT1
uniform.Gumbel	NN0T1	CF2T5	RKG	CF2T1	BGT5	BGT1
uniform.Galambos	CF2T5	NN1T1	CF2T1	BGT5	BBG	RKG
uniform.HuslerReiss	NN0T1	CF2T5	CF2T1	RKG	BBG	BGT1
uniform.Joe	CF2T5	CF2T1	NN0T1	RKG	BBG	BGT5
mix.Clayton.Gumbel	CF2T1	BallT1	BBG	BFT1	CF2T5	BGT1
mix.uniform.Frank	CF1T5	BallT1	NN0T5	BGT1	CF2T5	BBG
mix.Clayton.Frank	BFT5	BRG	qCvMG	BallT1	BGT1	qADG
mix.Frank.Gumbel	CvMT1	qADG	BGT1	BallT1	KSDG	BFT5
mix.Frank.Joe	qADG	NN0T1	BGT1	KSDG	BallT1	BFT1

normal.normal-shift-one.marginal	RKG	NN0T1	BGT5	BRG	CF2T1	BGT1
normal.normal-shift-both.marginal	CF4T1	NN0T1	CF2T5	BGT1	RKG	BGT5
normal.normal-stretch-one.marginal	NN0T5	NN1T1	CF1T1	CF2T5	CF4T1	CF2T1
normal.normal-stretch-both.marginal	BBG	NN1T5	NN0T5	NN1T1	CF2T5	CF2T1
normal.stretch-shift-one.marginal	CF4T5	BBG	CF4T1	NN1T1	CF2T5	NN0T1
normal.stretch-shift-two.marginal	BBG	NN0T5	CF4T1	CF2T5	NN1T1	CF2T1
uniform.rotated-uniform.marginal	CF2T1	CvMT1	BGT5	BGT1	RKG	KSDG
uniform.beta-one.marginal	CF4T1	CF4T5	CF1T5	NN1T1	CF2T1	CF2T5
uniform.beta-two.marginal	CF2T1	NN1T5	NN0T5	CF1T5	CF2T5	qADG
uni-exp-1.uni-exp-l.marginal	CF4T1	CF4T5	CF1T5	NN0T1	CF2T1	CF2T5
exp-exp-1.exp-exp-l.marginal	CF4T1	CF4T5	NN1T5	CF2T5	CF2T1	BBG
beta-nor-1.beta-nor-mean.marginal	CF4T5	NN0T1	BBG	CF2T1	qADG	CF2T5
beta-nor-1.beta-nor-sd.marginal	NN1T5	BBG	NN1T1	CF2T1	CF2T5	qADG
beta-beta-2.beta-beta-a.marginal	NN0T5	NN0T1	CF2T1	BBG	CF2T5	qADG
beta05.normal.marginal	CvMT1	BallT1	NN0T1	BFT1	BFT5	BBG
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uniform.uniform-diagonal-n	BGT1					
uniform.uniform-diagonal-b	BGT5					
normal-ind.normal-cor	BRG					
t-ind.t-cor	BGT1					
uniform.Frank	BBG					
uniform.Clayton	BGT5					
uniform.Gumbel	BBG					
uniform.Galambos	BGT1					
uniform.HuslerReiss	BGT5					
uniform.Joe	BGT1					
mix.Clayton.Gumbel	KSDG					
mix.uniform.Frank	BFT1					
mix.Clayton.Frank	KSDG					
mix.Frank.Gumbel	BFT1					
mix.Frank.Joe	BGT5					
normal.normal-shift-one.marginal	CF2T5					
normal.normal-shift-both.marginal	CF2T1					
normal.normal-stretch-one.marginal	NN0T1					
normal.normal-stretch-both.marginal	NN0T1					
normal.stretch-shift-one.marginal	CF2T1					
normal.stretch-shift-two.marginal	NN0T1					
uniform.rotated-uniform.marginal	BBG					
uniform.beta-one.marginal	BBG					
uniform.beta-two.marginal	BBG					
uni-exp-1.uni-exp-l.marginal	BBG					
exp-exp-1.exp-exp-l.marginal	BRG					
beta-nor-1.beta-nor-mean.marginal	KSDG					
beta-nor-1.beta-nor-sd.marginal	KSDG					
beta-beta-2.beta-beta-a.marginal	KSDG					
beta05.normal.marginal	KSDG					

Table 29: Power, continuous data, gof+hybrid methods, D=2, no estimation

11.1.9 Continuous data, gof+hybrid methods, no estimation, Dim=5

uniform.uniform-diagonal-n	qCvMG	MMDT5	ADT5	NN5T5	CvMT5	BRG
uniform.uniform-diagonal-b	MMDT5	qCvMG	ADT5	MMDT1	NN5T5	BRG
normal-ind.normal-corA	qCvMG	ADT5	CvMT5	qKSG	ADT1	CvMT1
normal-ind.normal-corB	qCvMG	ADT5	CvMT5	qKSG	ADT1	CvMT1
t-ind.t-cor	qCvMG	ADT5	CvMT5	qKSG	ADT1	CvMT1
uniform.Frank	qCvMG	ADT5	MMDT5	CvMT5	AZT5	qKSG
uniform.Clayton	qCvMG	BRG	MMDT1	CvMT5	ADT5	NN5T5
uniform.Gumbel	qCvMG	MMDT5	ADT5	CvMT5	qKSG	AZT5
uniform.Joe	qCvMG	MMDT5	qKSG	CvMT5	NN5T5	ADT5
mix.uniform.Clayton	MMDT5	AZT5	qCvMG	CvMT5	qKSG	qADG
mix.Clayton.Gumbel	MMDT5	AZT5	MMDT1	CvMT5	ADT5	qKSG
mix.uniform.Frank	MMDT5	qCvMG	AZT5	qKSG	CvMT5	qKG

mix.Clayton.Frank	MMDT5	NN5T5	MMDT1	NN5T1	CF3T5	NN1T1
mix.Frank.Gumbel	MMDT5	CF1T5	MMDT1	NN5T5	NN5T1	AZT5
mix.Frank.Joe	MMDT5	AZT5	MMDT1	ADT5	CvMT5	qKSG
normal.normal-shift-one.marginal	BFT5	AZT5	MMDT5	BFT1	AZT1	MMDT1
normal.shift-all.marginal	qKSG	qCvMG	AZT1	BFT1	MMDT1	KST5
normal.normal-stretch-one.marginal	BGT5	MMDT5	AZT5	BGT1	CF1T5	BallT1
normal.stretch-all.marginal	BGT1	BallT1	AZT5	BGT5	MMDT5	BFT5
normal.stretch-shift-one.marginal	BFT5	AZT5	MMDT5	BFT1	AZT1	MMDT1
normal.stretch-shift-all.marginal	qKSG	qCvMG	KST5	AZT5	BFT5	MMDT5
uniform.rotated-uniform.marginal	BFT5	MMDT5	AZT5	BFT1	MMDT1	AZT1
uniform.beta-one.marginal	BRG	KST5	AZT5	BFT5	MMDT5	CvMT5
uniform.beta-two.marginal	BGT1	BGT5	BRG	BallT1	MMDT5	AZT5
uniform.all-normal.marginal	AZT1	BGT1	MMDT1	BallT1	AZT5	BFT5
beta22.all-normal.marginal	BGT1	BallT1	AZT5	BGT5	MMDT5	AZT1
uniform.all-beta.marginal	BGT5	BGT1	BRG	BallT1	MMDT5	KT5
uniform.all-exp.marginal	BGT1	AZT5	BGT5	MMDT5	BallT1	BFT5
uniform.all-gamma.marginal	BGT1	BallT1	AZT5	BGT5	MMDT5	BFT5
uniform.different.marginal	BGT5	BGT1	BallT1	AZT5	MMDT5	KT5
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uniform.uniform-diagonal-n	MMDT1	ADT1	CF3T5	qKSG	CF4T5	CvMT1
uniform.uniform-diagonal-b	CvMT5	ADT1	qADG	CF3T5	AZT5	CvMT1
normal-ind.normal-corA	KST5	qADG	qKG	AZT5	NN5T5	MMDT5
normal-ind.normal-corB	KST5	qKG	qADG	MMDT5	NN5T5	AZT5
t-ind.t-cor	KST5	qKG	NN5T5	KST1	AZT5	MMDT5
uniform.Frank	MMDT1	BRG	NN5T5	ADT1	CvMT1	KST5
uniform.Clayton	AZT5	BFT5	MMDT5	ADT1	AZT1	qADG
uniform.Gumbel	NN5T5	ADT1	KST5	CvMT1	qKG	MMDT1
uniform.Joe	AZT5	KST5	MMDT1	qKG	CvMT1	CF3T5
mix.uniform.Clayton	ADT5	KST5	BRG	qKG	MMDT1	BFT5
mix.Clayton.Gumbel	KST5	qCvMG	qKG	AZT1	ADT1	CvMT1
mix.uniform.Frank	ADT5	KST5	MMDT1	BRG	KT5	CvMT1
mix.Clayton.Frank	NN1T5	CF1T1	NN0T5	CF4T5	CF1T5	CF4T1
mix.Frank.Gumbel	CF4T1	CF2T1	CF2T5	CF4T5	ADT5	CvMT5
mix.Frank.Joe	NN5T1	NN5T5	qKG	qCvMG	KST5	KT5
normal.normal-shift-one.marginal	BallT1	KST5	qKSG	CvMT5	qKG	KT5
normal.shift-all.marginal	CvMT5	AZT5	BFT5	MMDT5	CvMT1	ADT5
normal.normal-stretch-one.marginal	BFT5	NN5T5	qKG	KT5	qADG	MMDT1
normal.stretch-all.marginal	AZT1	MMDT1	CF1T5	BFT1	KT5	CF2T5
normal.stretch-shift-one.marginal	BallT1	KST5	qKSG	CvMT5	qKG	KT5
normal.stretch-shift-all.marginal	qKG	CvMT5	BallT1	AZT1	BFT1	BGT5
uniform.rotated-uniform.marginal	BRG	BallT1	NN5T5	KT5	CF1T5	NN5T1
uniform.beta-one.marginal	qCvMG	qKSG	ADT5	BallT1	KT5	BFT1
uniform.beta-two.marginal	KT5	KST5	MMDT1	qKG	qKSG	AZT1
uniform.all-normal.marginal	BGT5	MMDT5	KT5	BFT1	KST5	CF2T1
beta22.all-normal.marginal	MMDT1	BFT5	KT5	BFT1	KST5	CF2T1
uniform.all-beta.marginal	KST5	AZT5	qKSG	qKG	MMDT1	AZT1
uniform.all-exp.marginal	AZT1	KT5	MMDT1	KST5	BFT1	qKG
uniform.all-gamma.marginal	MMDT1	AZT1	KT5	BFT1	CF1T5	KST5
uniform.different.marginal	qKG	KST5	CF1T5	qKSG	BFT5	MMDT1
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uniform.uniform-diagonal-n	NN1T5	AZT5	NN0T5	CF2T5	NN5T1	qADG
uniform.uniform-diagonal-b	CF4T5	NN1T5	qKSG	NN0T5	CF2T5	NN5T1
normal-ind.normal-corA	NN5T1	KST1	CF3T5	MMDT1	NN1T5	AZT1
normal-ind.normal-corB	NN5T1	KST1	CF3T5	MMDT1	CF4T5	AZT1
t-ind.t-cor	CF3T5	NN5T1	NN1T5	qADG	KT5	CF4T5
uniform.Frank	qKG	NN5T1	AZT1	BFT5	CF3T5	KST1
uniform.Clayton	qKSG	NN5T1	CvMT1	CF3T5	CF4T5	NN0T5
uniform.Gumbel	BRG	NN5T1	CF3T5	BFT5	KST1	NN1T5
uniform.Joe	KST1	ADT1	NN5T1	NN1T5	NN0T5	CF4T5
mix.uniform.Clayton	KT5	AZT1	CvMT1	NN5T5	ADT1	NN5T1
mix.Clayton.Gumbel	KT5	NN5T1	NN5T5	qADG	CF1T5	KST1
mix.uniform.Frank	AZT1	ADT1	KST1	NN5T5	NN5T1	BFT5
mix.Clayton.Frank	qKG	KT5	NN0T1	CF2T5	CF2T1	KT1
mix.Frank.Gumbel	qKSG	KT5	qKG	KST5	CF1T1	ADT1
mix.Frank.Joe	ADT1	CF1T5	BRG	AZT1	CvMT1	CF4T1
normal.normal-shift-one.marginal	KST1	CvMT1	KT1	qCvMG	NN5T5	ADT1
normal.shift-all.marginal	qKG	BallT1	ADT1	KST1	KT5	NN5T5

normal.normal-stretch-one.marginal	CF4T5	CF2T5	NN5T1	CF3T5	KT1	AZT1
normal.stretch-all.marginal	CF4T5	KST5	qKG	CF2T1	CF4T1	qKSG
normal.stretch-shift-one.marginal	KST1	BGT5	CvMT1	NN5T5	KT1	qCvMG
normal.stretch-shift-all.marginal	MMDT1	KT5	CvMT1	KST1	BGT1	ADT5
uniform.rotated-uniform.marginal	qKG	CF3T5	ADT5	CF4T5	NN1T5	CF2T5
uniform.beta-one.marginal	CvMT1	AZT1	MMDT1	KST1	BGT5	ADT1
uniform.beta-two.marginal	CF4T1	CF2T1	CF4T5	CF2T5	KT1	KST1
uniform.all-normal.marginal	CF4T1	CF2T5	CF4T5	qKSG	qKG	NN5T5
beta22.all-normal.marginal	CF4T1	CF2T5	CF4T5	NN5T5	qKG	qKSG
uniform.all-beta.marginal	KT1	CF4T1	CF2T1	KST1	CF4T5	CF2T5
uniform.all-exp.marginal	qKSG	KT1	NN5T5	KST1	CvMT5	CF2T1
uniform.all-gamma.marginal	qKG	CF2T5	CF4T5	qKSG	qADG	CF2T1
uniform.different.marginal	CF4T5	CF2T5	qCvMG	AZT1	qADG	CvMT5
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	KST5	qKG	CF1T5	AZT1	CF1T1	CF4T1
uniform.uniform-diagonal-b	AZT1	KST5	CF1T5	CF1T1	qKG	CF4T1
normal-ind.normal-corA	BFT5	KT5	CF4T5	NN0T5	NN0T1	NN1T1
normal-ind.normal-corB	NN1T1	NN0T5	KT5	CF1T1	NN1T5	NN0T1
t-ind.t-cor	KT1	AZT1	NN0T5	NN1T1	CF1T1	BFT5
uniform.Frank	CF4T5	NN0T5	CF1T1	NN1T5	NN1T1	CF4T1
uniform.Clayton	CF2T5	KST5	NN1T5	qKG	BallT1	BFT1
uniform.Gumbel	NN0T5	AZT1	CF4T5	KT5	CF1T1	NN0T1
uniform.Joe	AZT1	BFT5	KT5	BRG	CF2T5	NN0T1
mix.uniform.Clayton	KST1	BallT1	BGT5	KT1	CF1T1	CF3T5
mix.Clayton.Gumbel	BRG	KT1	BFT5	CF1T1	CF4T1	CF3T5
mix.uniform.Frank	KT1	BallT1	BGT5	CF3T5	CF1T1	BFT1
mix.Clayton.Frank	AZT5	AZT1	KST5	BRG	ADT1	BGT5
mix.Frank.Gumbel	CF3T5	qCvMG	CvMT1	NN1T1	NN1T5	KST1
mix.Frank.Joe	CF2T1	CF1T1	CF4T5	KT1	CF3T5	CF2T5
normal.normal-shift-one.marginal	NN5T1	ADT5	CF3T5	NN1T5	NN0T5	CF1T1
normal.shift-all.marginal	KT1	NN5T1	CF3T5	CF1T5	NN1T1	CF1T1
normal.normal-stretch-one.marginal	CF4T1	NN1T5	ADT5	KST5	qKSG	CF2T1
normal.stretch-all.marginal	qADG	KT1	qCvMG	NN5T5	KST1	NN5T1
normal.stretch-shift-one.marginal	NN5T1	CF1T5	ADT1	BGT1	CF3T5	NN1T1
normal.stretch-shift-all.marginal	ADT1	KT1	CF1T5	NN5T5	NN5T1	CF2T5
uniform.rotated-uniform.marginal	qKSG	KST5	NN0T5	CF1T1	qCvMG	KT1
uniform.beta-one.marginal	qKG	BGT1	KT1	NN5T5	NN5T1	CF3T5
uniform.beta-two.marginal	CvMT5	BFT5	NN5T5	NN0T1	qCvMG	NN5T1
uniform.all-normal.marginal	KT1	NN0T1	NN5T1	KST1	CvMT5	NN0T5
beta22.all-normal.marginal	NN5T1	CvMT5	KT1	KST1	NN0T1	NN0T5
uniform.all-beta.marginal	BFT5	BFT1	qCvMG	NN5T5	NN5T1	NN0T1
uniform.all-exp.marginal	CF4T1	CF2T5	CF4T5	NN5T1	NN0T1	qCvMG
uniform.all-gamma.marginal	CF4T1	KT1	NN5T5	qCvMG	KST1	ADT5
uniform.different.marginal	ADT5	KT1	CF2T1	CF4T1	KST1	NN5T5
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	KST1	NN0T1	NN1T1	CF2T1	BFT5	KT5
uniform.uniform-diagonal-b	KST1	NN0T1	BFT5	NN1T1	CF2T1	BFT1
normal-ind.normal-corA	CF1T1	CF1T5	CF2T5	CF4T1	BFT1	KT1
normal-ind.normal-corB	CF4T1	BFT5	CF1T5	CF2T5	BFT1	KT1
t-ind.t-cor	MMDT1	CF4T1	NN0T1	CF2T5	CF1T5	BFT1
uniform.Frank	CF1T5	CF2T5	BFT1	NN0T1	qADG	KT5
uniform.Clayton	CF4T1	CF2T1	CF1T1	NN0T1	KST1	NN1T1
uniform.Gumbel	CF2T5	NN1T1	CF4T1	CF1T5	qADG	BallT1
uniform.Joe	NN1T1	CF1T5	CF4T1	CF1T1	KT1	BallT1
mix.uniform.Clayton	CF4T1	NN0T5	NN0T1	CF4T5	CF2T1	NN1T5
mix.Clayton.Gumbel	CF2T1	CF4T5	BallT1	CF2T5	BGT5	NN1T5
mix.uniform.Frank	NN0T1	CF4T5	NN0T5	CF4T1	NN1T5	NN1T1
mix.Clayton.Frank	qADG	ADT5	qKSG	KST1	BFT1	CvMT5
mix.Frank.Gumbel	KT1	BRG	AZT1	BGT5	NN0T5	BGT1
mix.Frank.Joe	KST1	NN1T1	NN1T5	NN0T5	BGT5	BFT5
normal.normal-shift-one.marginal	NN1T1	CF4T5	CF1T5	CF4T1	CF2T5	NN0T1
normal.shift-all.marginal	NN1T5	CF4T5	CF4T1	NN0T5	CF2T5	BGT5
normal.normal-stretch-one.marginal	CF1T1	KST1	qCvMG	CvMT1	ADT1	NN1T1
normal.stretch-all.marginal	CvMT5	ADT5	CF3T5	CF1T1	NN1T1	CvMT1
normal.stretch-shift-one.marginal	CF4T5	CF1T1	ADT5	NN1T5	NN0T5	CF4T1
normal.stretch-shift-all.marginal	CF4T5	CF4T1	CF2T1	CF3T5	CF1T1	NN1T5
uniform.rotated-uniform.marginal	NN1T1	BGT5	CF4T1	CvMT5	NN0T1	KST1

uniform.beta-one.marginal	CF4T5	CF4T1	CF1T1	NN1T5	NN0T5	CF2T5
uniform.beta-two.marginal	CF3T5	BFT1	NN0T5	CvMT1	NN1T5	CF1T1
uniform.all-normal.marginal	CF3T5	qCvMG	CF1T1	NN1T5	NN1T1	CvMT1
beta22.all-normal.marginal	CF3T5	qCvMG	CF1T1	NN1T5	NN1T1	CvMT1
uniform.all-beta.marginal	CvMT5	NN0T5	CF3T5	CvMT1	NN1T1	CF1T1
uniform.all-exp.marginal	NN0T5	CF3T5	CF1T1	CvMT1	NN1T5	NN1T1
uniform.all-gamma.marginal	NN5T1	CvMT5	CF3T5	NN1T5	CF1T1	CvMT1
uniform.different.marginal	NN5T1	BFT1	CF3T5	CvMT1	CF1T1	NN1T5
—	—	—	—	—	—	—
uniform.uniform-diagonal-n	KT1	BFT1	BallT1	BGT5	BGT1	
uniform.uniform-diagonal-b	KT5	KT1	BGT5	BallT1	BGT1	
normal-ind.normal-corA	CF2T1	BGT5	BallT1	BRG	BGT1	
normal-ind.normal-corB	CF2T1	BallT1	BGT5	BGT1	BRG	
t-ind.t-cor	CF2T1	BGT5	BallT1	BGT1	BRG	
uniform.Frank	CF2T1	BallT1	KT1	BGT1	BGT5	
uniform.Clayton	CF1T5	KT5	KT1	BGT5	BGT1	
uniform.Gumbel	CF2T1	BFT1	KT1	BGT5	BGT1	
uniform.Joe	BFT1	CF2T1	qADG	BGT5	BGT1	
mix.uniform.Clayton	NN1T1	CF2T5	BFT1	BGT1	CF1T5	
mix.Clayton.Gumbel	BFT1	NN1T1	BGT1	NN0T5	NN0T1	
mix.uniform.Frank	BGT1	CF2T5	CF2T1	CF1T5	qADG	
mix.Clayton.Frank	qCvMG	CvMT1	BallT1	BFT5	BGT1	
mix.Frank.Gumbel	BFT5	BallT1	qADG	BFT1	NN0T1	
mix.Frank.Joe	BallT1	BGT1	NN0T1	BFT1	qADG	
normal.normal-shift-one.marginal	CF2T1	BGT1	BGT5	BRG	qADG	
normal.shift-all.marginal	CF2T1	NN0T1	BGT1	BRG	qADG	
normal.normal-stretch-one.marginal	NN0T5	CvMT5	BFT1	NN0T1	BRG	
normal.stretch-all.marginal	NN1T5	ADT1	BRG	NN0T5	NN0T1	
normal.stretch-shift-one.marginal	CF2T5	BRG	CF2T1	NN0T1	qADG	
normal.stretch-shift-all.marginal	NN1T1	NN0T5	BRG	NN0T1	qADG	
uniform.rotated-uniform.marginal	ADT1	CF2T1	CvMT1	BGT1	qADG	
uniform.beta-one.marginal	NN0T1	CF2T1	NN1T1	qADG	CF1T5	
uniform.beta-two.marginal	NN1T1	ADT1	ADT5	CF1T5	qADG	
uniform.all-normal.marginal	ADT1	ADT5	BRG	CF1T5	qADG	
beta22.all-normal.marginal	ADT1	ADT5	CF1T5	BRG	qADG	
uniform.all-beta.marginal	NN1T5	ADT1	ADT5	CF1T5	qADG	
uniform.all-exp.marginal	ADT1	ADT5	CF1T5	BRG	qADG	
uniform.all-gamma.marginal	NN1T1	ADT1	NN0T5	BRG	NN0T1	
uniform.different.marginal	NN1T1	ADT1	BRG	NN0T5	NN0T1	

Table 30: Power, continuous data, gof+hybrid methods, D=5, no estimation

11.1.10 Discrete data, no estimation

uniform.uniform-diagonal-n	P	KL	TV	H	AD	KS	CvM	K
uniform.uniform-diagonal-b	P	KL	H	TV	AD	CvM	KS	K
normal-ind.normal-cor	KL	H	P	TV	AD	CvM	KS	K
t-ind.t-cor	H	KL	TV	P	AD	CvM	KS	K
uniform.Frank	P	KL	H	TV	AD	CvM	KS	K
uniform.Clayton	KL	P	H	TV	AD	CvM	K	KS
uniform.Gumbel	P	KL	H	TV	AD	CvM	KS	K
uniform.Galambos	KL	H	P	TV	AD	CvM	KS	K
uniform.HuslerReiss	P	KL	H	TV	AD	CvM	K	KS
uniform.Joe	KL	H	P	TV	AD	CvM	KS	K
mix.Clayton.Gumbel	P	TV	KL	H	K	AD	CvM	KS
mix.uniform.Frank	KL	H	TV	P	K	AD	CvM	KS
mix.Clayton.Frank	P	KL	TV	H	K	CvM	AD	KS
mix.Frank.Gumbel	P	TV	KL	H	K	CvM	AD	KS
mix.Frank.Joe	P	TV	KL	H	K	AD	CvM	KS
normal.normal-shift-one.marginal	KS	AD	CvM	K	KL	TV	P	H
normal.normal-shift-both.marginal	CvM	AD	KS	KL	P	TV	K	H
normal.normal-stretch-one.marginal	P	KL	K	TV	AD	H	KS	CvM
normal.normal-stretch-both.marginal	P	KL	K	TV	AD	CvM	KS	H

normal.stretch-shift-one.marginal	KS	AD	CvM	P	KL	K	TV	H
normal.stretch-shift-two.marginal	AD	CvM	KS	P	K	KL	TV	H
uniform.rotated-uniform.marginal	K	KL	P	H	TV	KS	CvM	AD
uniform.beta-one.marginal	CvM	AD	KS	K	H	KL	P	TV
uniform.beta-two.marginal	K	KL	P	H	TV	KS	AD	CvM
uni-exp-1.uni-exp-l.marginal	AD	CvM	KS	H	K	KL	TV	P
exp-exp-1.exp-exp-l.marginal	KS	AD	CvM	H	TV	K	KL	P
beta-nor-1.beta-nor-mean.marginal	K	KL	H	TV	P	AD	KS	CvM
beta-nor-1.beta-nor-sd.marginal	K	KL	TV	H	AD	P	CvM	KS
beta-beta-2.beta-beta-a.marginal	H	K	KL	AD	P	TV	CvM	KS
beta05.normal.marginal	P	KL	TV	H	K	AD	KS	CvM

Table 31: Power, discrete data, no estimation

11.1.11 Discrete data, with estimation

normal-sigma.t	P	KL	AD	H	CvM	K	KS	TV
Frank.Frank	P	KL	H	TV	K	KS	CvM	AD
Clayton.Clayton	P	KL	H	TV	K	KS	AD	CvM
Gumbel.Gumbel	P	KL	TV	H	K	KS	AD	CvM
Galambos.Galambos	KL	H	TV	P	K	KS	AD	CvM
HuslerReiss.HuslerReiss	TV	KL	H	P	K	KS	AD	CvM
Joe.Joe	TV	KL	P	H	K	AD	KS	CvM
mix.uniform.Frank	P	KL	TV	H	AD	K	KS	CvM
mix.Clayton.Frank	H	KL	TV	P	K	KS	CvM	AD
mix.Clayton.Clayton	TV	KL	P	H	K	KS	CvM	AD
mix.uniform.Joe	TV	KL	P	H	AD	KS	K	CvM
mix.uniform.Plackett	P	KL	H	TV	AD	CvM	KS	K
Clayton.marginal.normal-sigma	KS	K	CvM	AD	TV	KL	H	P
Gumbel.marginal.normal-sigma	KS	K	CvM	AD	TV	KL	H	P
Joe.marginal.exponential	KS	K	CvM	AD	TV	KL	H	P
normal-mean.t.marginal	P	KL	AD	CvM	K	H	KS	TV
Clayton.marginal.normal.marginal	AD	P	K	KL	TV	H	CvM	KS
Joe.marginal.exponential.marginal	KS	AD	CvM	K	TV	H	KL	P
Plackett.marginal.beta22.marginal	K	H	KL	AD	TV	CvM	P	KS
Frank.marginal.dblexp.marginal	K	AD	KL	H	TV	CvM	KS	P
Frank.marginal.linear.marginal	KS	AD	K	CvM	H	KL	TV	P
Joe.marginal.truncexp.marginal	K	KS	KL	H	TV	P	CvM	AD
Clayton.marginal.betaa1.marginal	H	KL	K	TV	KS	AD	CvM	P
beta22.normal-mean.marginal	K	KL	H	TV	AD	P	CvM	KS
beta22.normal-sd.marginal	K	KL	H	TV	P	AD	KS	CvM
beta22.lognormal-mean.marginal	K	KL	H	TV	AD	KS	CvM	P
beta22.lognormal-sd.marginal	K	TV	KL	H	AD	KS	CvM	P
beta22.exponential.marginal	K	TV	KL	H	AD	CvM	KS	P
exp1.normal-sd.marginal	KS	K	CvM	AD	TV	KL	H	P
beta05.gamma.marginal	K	TV	KS	KL	H	AD	CvM	P

Table 32: Power, discrete data, with estimation

11.2 Two-sample

11.2.1 Continuous data, D=2

NormalD2	ES	EP	AZ	MMD	NN5	AD	BF	CF1	CvM
tD2	EP	ES	AZ	NN5	MMD	AD	CvM	BF	CF1
UniformMixtureD2	MMD	AZ	ES	NN5	EP	CF1	CF4	CF2	NN1
FrankD2	MMD	AZ	EP	ES	NN5	BF	AD	Ball	CvM
ClaytonD2	NN5	AZ	BF	MMD	ES	EP	CF1	CF4	Ball
GumbelD2	AZ	ES	MMD	EP	NN5	AD	BF	CvM	Ball
GalambosD2	AZ	MMD	ES	EP	NN5	BF	AD	CvM	Ball

HuslerReissD2	NN5	AZ	MMD	ES	EP	BF	Ball	CF1	CF4
ClaytonGumbelD2	MMD	NN5	AZ	ES	EP	CF1	CF4	CF2	NN0
UniformFrankD2	ES	EP	NN5	MMD	AZ	CF1	CF4	NN1	K
ParetoSimplexD2	AZ	BF	MMD	Ball	AD	CvM	BG	KS	EP
KhoudrajiClaytonD2	ES	EP	MMD	NN5	AZ	CF1	CF4	NN1	CF2
NormalUniformD2	MMD	AZ	ES	EP	NN5	BF	AD	Ball	CvM
JoeD2	MMD	AZ	ES	EP	NN5	BF	CF1	AD	CvM
DalitzCleoD2	MMD	NN5	ES	CvM	AZ	AD	CF1	CF4	NN1
DalitzPDGD2	MMD	NN5	CF1	AZ	ES	CF4	NN1	CvM	AD
DalitzBabarD2	MMD	NN5	AZ	CvM	AD	CF1	NN1	ES	CF4
NormalShiftM	MMD	BF	AZ	Ball	KS	CvM	AD	K	ES
NormalStretchM	BG	Ball	ES	AZ	MMD	K	NN5	EP	AD
UniformRotateM	MMD	BF	AZ	Ball	NN5	K	ES	CF1	CF4
UniformBetaM	BG	Ball	AZ	MMD	K	AD	ES	NN5	BF
TruncExponentialM	CvM	BF	AD	Ball	MMD	AZ	KS	BG	ES
DalitzCleoM	MMD	NN5	AZ	AD	EP	CF1	CvM	BG	CF4
DalitzPDGM	MMD	AZ	BG	NN5	ES	AD	Ball	EP	CF1
DalitzBabarM	MMD	AZ	AD	BG	EP	NN5	Ball	CvM	BF
FrankExponentialM	BF	AZ	MMD	KS	K	CvM	AD	ES	Ball
FrankLinearM	BF	AZ	MMD	Ball	KS	K	AD	CvM	ES
FrankNormalM	K	ES	BG	NN5	EP	MMD	AD	AZ	KS
ClaytonExponentialM	EP	AZ	K	BF	MMD	NN5	ES	KS	CF1
ClaytonLinearM	AZ	ES	MMD	EP	NN5	BF	K	CF1	KS
ClaytonNormalM	NN5	MMD	ES	K	CF1	AZ	EP	CF4	NN1
GalambosExponentialM	NN5	AZ	ES	MMD	EP	K	CF1	BF	CF4
GalambosLinearM	AZ	MMD	ES	EP	BF	NN5	K	KS	CF1
GalambosNormalM	NN5	ES	MMD	CF1	EP	K	AZ	CF4	NN1
CheckerboardD2	NN5	MMD	EP	CF1	ES	CF4	NN1	CF2	NN0
TentD2	NN5	MMD	ES	EP	CF1	CF4	NN1	CF2	AZ

NormalD2	CF4	K	NN1	KS	CF2	NN0	Ball	BG
tD2	KS	K	CF4	CF2	NN1	NN0	Ball	BG
UniformMixtureD2	AD	NN0	BF	CvM	Ball	K	KS	BG
FrankD2	CF1	KS	K	CF4	NN1	NN0	CF2	BG
ClaytonD2	CF2	NN1	AD	NN0	K	CvM	KS	BG
GumbelD2	CF1	KS	CF4	NN1	K	CF2	NN0	BG
GalambosD2	CF1	KS	CF4	NN1	K	CF2	NN0	BG
HuslerReissD2	AD	CF2	NN1	CvM	K	NN0	KS	BG
ClaytonGumbelD2	NN1	AD	BF	CvM	K	KS	Ball	BG
UniformFrankD2	CF2	AD	CvM	KS	NN0	Ball	BF	BG
ParetoSimplexD2	ES	NN5	CF1	CF4	K	NN1	CF2	NN0
KhoudrajiClaytonD2	Ball	BF	K	AD	NN0	CvM	KS	BG
NormalUniformD2	CF1	CF4	NN1	KS	K	CF2	NN0	BG
JoeD2	Ball	CF4	NN1	K	KS	CF2	NN0	BG
DalitzCleoD2	CF2	EP	KS	BF	NN0	Ball	K	BG
DalitzPDGD2	CF2	EP	KS	NN0	BF	Ball	K	BG
DalitzBabarD2	CF2	KS	BF	EP	Ball	NN0	K	BG
NormalShiftM	EP	NN5	CF1	CF4	NN1	BG	NN0	CF2
NormalStretchM	BF	CvM	KS	CF1	CF4	NN0	CF2	NN1
UniformRotateM	NN1	EP	NN0	CF2	AD	KS	CvM	BG
UniformBetaM	KS	EP	CvM	CF1	CF4	NN1	CF2	NN0
TruncExponentialM	K	EP	NN5	CF1	NN1	CF4	NN0	CF2
DalitzCleoM	ES	NN1	KS	Ball	CF2	K	NN0	BF
DalitzPDGM	CF4	CvM	NN1	KS	BF	CF2	NN0	K
DalitzBabarM	KS	ES	CF1	NN1	CF4	CF2	K	NN0
FrankExponentialM	EP	BG	NN5	CF1	CF4	NN1	NN0	CF2
FrankLinearM	EP	NN5	BG	CF1	CF4	NN1	NN0	CF2
FrankNormalM	CF1	Ball	CvM	NN1	CF4	NN0	CF2	BF
ClaytonExponentialM	CvM	AD	Ball	CF4	NN1	BG	CF2	NN0
ClaytonLinearM	CF4	Ball	NN1	CF2	AD	NN0	CvM	BG
ClaytonNormalM	CF2	NN0	BG	KS	AD	Ball	BF	CvM
GalambosExponentialM	KS	NN1	CF2	NN0	CvM	AD	Ball	BG
GalambosLinearM	CF4	Ball	NN1	CF2	AD	CvM	NN0	BG
GalambosNormalM	CF2	NN0	BG	AD	KS	CvM	Ball	BF
CheckerboardD2	AZ	K	BF	AD	Ball	CvM	KS	BG
TentD2	NN0	K	Ball	KS	BF	AD	BG	CvM

Table 33: Power, continuous data, dim=2

11.2.2 Continuous data, D=5

NormalD5	AD	CvM	MMD	NN5	AZ	KS	CF1	CF4
tD5	AD	CvM	NN5	KS	AZ	MMD	CF1	CF4
FrankD5	MMD	AZ	NN5	CvM	AD	CF1	KS	NN1
ClaytonD5	MMD	AD	CvM	AZ	NN5	CF1	CF4	Ball
GumbelD5	MMD	AZ	AD	CvM	NN5	KS	CF1	CF4
JoeD5	CvM	NN5	AZ	BF	MMD	CF1	Ball	KS
UniformFrankD5	AD	CvM	MMD	NN5	AZ	KS	CF1	CF4
FrankClaytonD5	MMD	NN5	CF2	CF4	CF1	NN1	AD	AZ
FrankJoeD5	MMD	NN5	K	CF2	CF4	CF1	NN1	AD
UniformExponentialM5	BF	Ball	BG	AZ	MMD	CvM	KS	AD
FrankExponentialM5	BF	AZ	MMD	Ball	CvM	KS	BG	AD
FrankLinearM5	BF	AZ	MMD	Ball	AD	CvM	KS	BG
FrankNormalM5	BG	Ball	CF2	MMD	CF4	AZ	NN0	K
ClaytonExponentialM5	BF	AZ	MMD	BG	NN5	Ball	KS	K
ClaytonLinearM5	AZ	NN5	MMD	BF	Ball	CF1	CF4	CF2
ClaytonNormalM5	BG	CF2	CF4	MMD	NN0	NN5	AZ	K
SumModD5	NN5	CF4	CF2	CF1	NN1	MMD	NN0	AZ

NormalD5	NN1	CF2	BF	K	Ball	NN0	BG
tD5	NN1	K	CF2	BF	NN0	Ball	BG
FrankD5	CF4	BF	Ball	CF2	NN0	K	BG
ClaytonD5	CF2	NN1	KS	BF	NN0	K	BG
GumbelD5	CF2	Ball	BF	NN1	K	NN0	BG
JoeD5	AD	CF2	CF4	NN1	K	NN0	BG
UniformFrankD5	NN1	CF2	BF	Ball	K	NN0	BG
FrankClaytonD5	CvM	KS	K	BG	NN0	Ball	BF
FrankJoeD5	KS	CvM	AZ	Ball	NN0	BF	BG
UniformExponentialM5	K	NN5	CF4	CF1	NN1	CF2	NN0
FrankExponentialM5	K	NN5	CF1	NN1	CF4	CF2	NN0
FrankLinearM5	NN5	K	CF1	CF4	NN1	NN0	CF2
FrankNormalM5	NN5	KS	CF1	NN1	AD	CvM	BF
ClaytonExponentialM5	CvM	AD	CF1	CF4	NN1	CF2	NN0
ClaytonLinearM5	NN1	KS	CvM	AD	K	BG	NN0
ClaytonNormalM5	CF1	Ball	NN1	KS	BF	AD	CvM
SumModD5	AD	K	KS	BF	CvM	BG	Ball

Table 34: Power, continuous data, dim=5

11.2.3 Discrete data

NormalD2	Chisquare	AZ	BF	NN	AD	CvM	KS	K
tD2	Chisquare	AZ	NN	AD	CvM	BF	KS	K
UniformMixtureD2	Chisquare	NN	AD	BF	K	CvM	KS	AZ
FrankD2	Chisquare	BF	AD	NN	CvM	K	KS	AZ
ClaytonD2	BF	Chisquare	NN	K	AD	CvM	KS	AZ
GumbelD2	Chisquare	BF	AD	NN	CvM	KS	K	AZ
GalambosD2	Chisquare	AD	NN	BF	CvM	K	KS	AZ
HuslerReissD2	Chisquare	BF	NN	K	AD	CvM	KS	AZ
ClaytonGumbelD2	Chisquare	NN	AD	BF	CvM	K	KS	AZ
UniformFrankD2	Chisquare	NN	CvM	AD	K	KS	BF	AZ
ParetoSimplexD2	BF	AD	CvM	Chisquare	AZ	KS	NN	K
KhoudrajiClaytonD2	Chisquare	NN	BF	AD	CvM	K	KS	AZ
NormalUniformD2	Chisquare	NN	BF	AD	CvM	K	KS	AZ
JoeD2	Chisquare	BF	NN	K	AD	CvM	KS	AZ
DalitzCleoD2	Chisquare	NN	AD	CvM	KS	BF	K	AZ
DalitzPDGD2	Chisquare	NN	AD	CvM	BF	KS	AZ	K

DalitzBabarD2	Chisquare	NN	CvM	AD	KS	BF	AZ	K
NormalShiftM	AZ	BF	KS	AD	CvM	K	Chisquare	NN
NormalStretchM	Chisquare	AZ	K	NN	BF	AD	KS	CvM
UniformRotateM	BF	K	Chisquare	AZ	AD	KS	CvM	NN
UniformBetaM	K	Chisquare	AD	CvM	NN	KS	BF	AZ
TruncExponentialM	CvM	BF	AD	AZ	KS	Chisquare	K	NN
DalitzCleoM	Chisquare	AD	NN	CvM	K	KS	BF	AZ
DalitzPDGM	Chisquare	AD	CvM	K	NN	AZ	KS	BF
DalitzBabarM	Chisquare	AD	CvM	AZ	K	KS	NN	BF
FrankExponentialM	BF	AZ	K	KS	AD	CvM	Chisquare	NN
FrankLinearM	BF	AZ	KS	K	AD	CvM	Chisquare	NN
FrankNormalM	AZ	Chisquare	BF	K	NN	AD	KS	CvM
ClaytonExponentialM	BF	AZ	K	Chisquare	KS	NN	AD	CvM
ClaytonLinearM	Chisquare	BF	K	NN	KS	AZ	CvM	AD
ClaytonNormalM	Chisquare	BF	K	AZ	NN	AD	KS	CvM
GalambosExponentialM	Chisquare	BF	NN	K	AZ	KS	CvM	AD
GalambosLinearM	Chisquare	BF	NN	K	KS	AZ	AD	CvM
GalambosNormalM	Chisquare	AZ	K	BF	NN	AD	KS	CvM
CheckerboardD2	Chisquare	NN	BF	K	KS	AD	CvM	AZ
TentD2	Chisquare	NN	BF	K	CvM	KS	AD	AZ

Table 35: Power, discrete data

Competing interests

The author is the author and maintainer of the R packages *MDgof* and *MD2sample*, which implement most of the methods compared here, and chose the case studies. The author received no funding for this work.

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